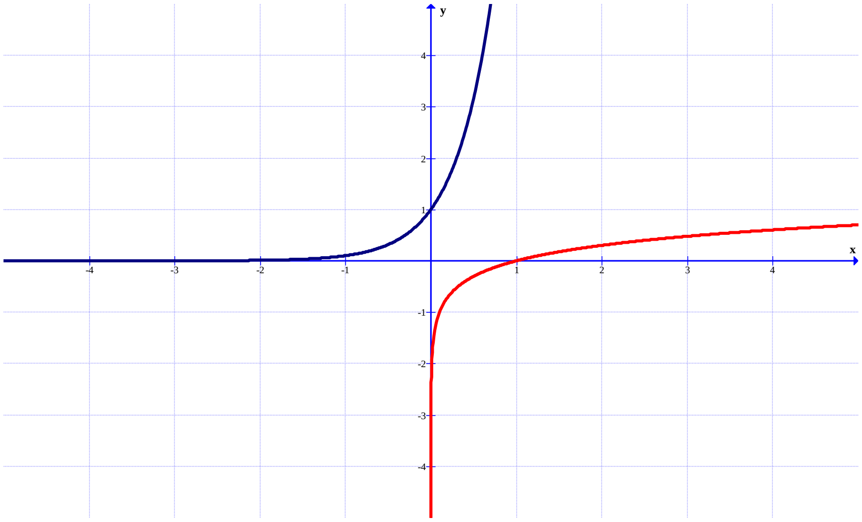
Pre-Calculus 30 **8.1 Understanding Logarithms (Day 1)**

Recall: to find the inverse of a function or relation, interchange the *x* and *y* values. The graphs of inverse functions/relations are reflections of each other in the line $y = x$.

$y = 10^x$



$x = 10^y$ or

$\log_c x = y$ is the ***inverse*** of the exponential function $c^x = y$. If you graph $y = c^x$ and then reflect it in the line $y = x$, you will get $y = \log_c x$. ($c > 0$ and $c \neq 1$)

A **common logarithm** is a logarithm with base 10. In this case, we don't write the base of the logarithm. The log button on a calculator is for **common logarithms.**

- $\log_{10} 100 = 2$ can be written as

Exponential Form	⟶	**Logarithmic Form**
$c^y = x$		$\log_c x = y$

Complete:

Exponential	Logarithmic
$7^4 = 2401$	
$5^2 = 25$	
	$\log_5 78\,125 = 7$
	$\log_{\frac{1}{2}} 16 = -4$
$p^q = r$	
	$\log_j h = m$

Evaluate:

a) $\log_2 8$

b) $\log_5 625$

c) $\log_6 1$

d) $\log_2 \sqrt{8}$

e) $\log_2 32$

f) $\log\ 1\,000\,000$

g) $\log_9 \sqrt[5]{81}$

h) $\log_3 9\sqrt{3}$

Restrictions on the value of 'c'

A logarithmic function is in the form $y = \log_c x$, where $c > 0$ and $c \neq 1$. Why does 'c' have these restrictions? (Hint: write in exponential form)

Helpful Hints

- $\log_c 1 = 0$ since in exponential form $c^0 = 1$.
- $\log_c c = 1$ since in exponential form $c^1 = c$
- $\log_c c^x = x$ since in exponential form $c^x = c^x$
- $c^{\log_c x} = x, x > 0$, since in logarithmic form $\log_c x = \log_c x$

Page 375 – Your Turn. Determine the value of *x*. Write in exponential form first.

1. $\log_4 x = -2$

2. $\log_{16} x = -\frac{1}{4}$

3. $\log_x 9 = \frac{2}{3}$

Assignment: page 380 #2 – 4, 12, 14a

Pre-Calculus 30 **8.1 Understanding Logarithms (Day 2)**

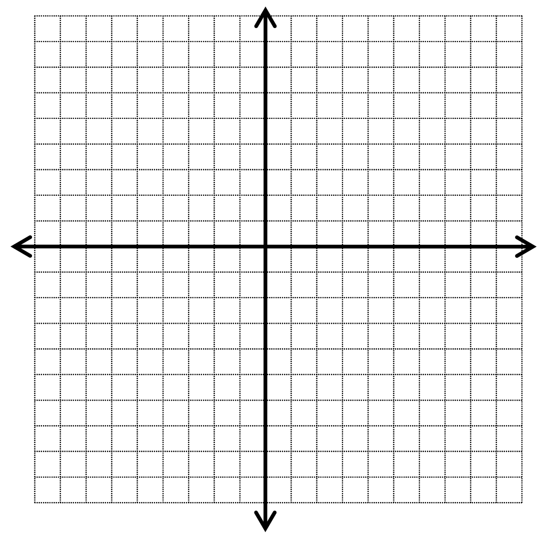
Recall:

- To find the ***inverse*** of a function, interchange x and y values.

Example

State the equation of the inverse of $f(x)=3^x$. State your answer in exponential and logarithmic form.

Sketch the graph of $f(x)=3^x$ and its inverse. Use a table of values for both. Note the relationship between the two.



Determine the following characteristics of the inverse graph:

Domain:

Range:

x-intercept, if it exists

y-intercept, if it exists

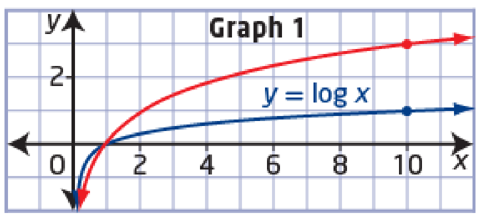
equations of any asymptotes

Assignment: Page 380 #1, 8, 10, 14b, 21 (try 24)

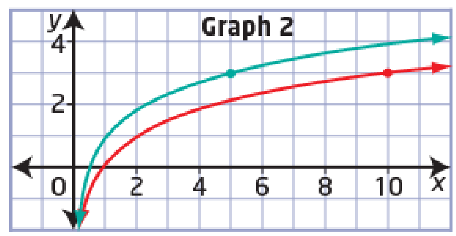
8.2 Transformations of Logarithmic Functions

Given the base function $y = \log_c x$, multiple transformations can be applied using the general transformation model $y = a\log_c\left(b(x-h)\right)+k$. The mapping notation for multiple transformations would be $(x,y)\rightarrow($________, ________$)$.

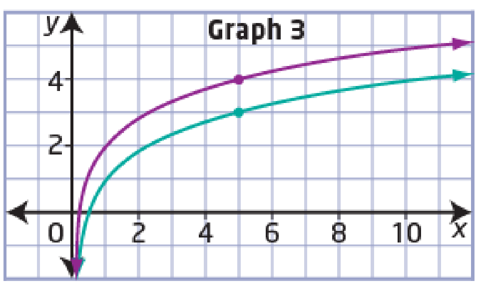
Example: The graphs below show how $y=\log x$ is transformed into $y = a\log\left(b(x-h)\right)+k$ by changing one parameter at a time. The effect on one key point is shown at each step. For each graph, explain the effect of the parameter introduced and write the equation of the transformed function.



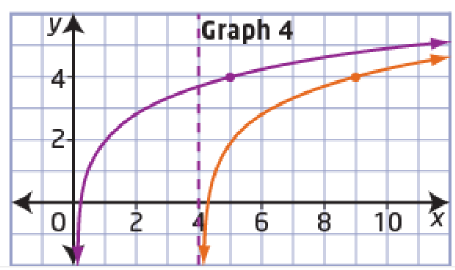
Graph 1:



Graph 2:



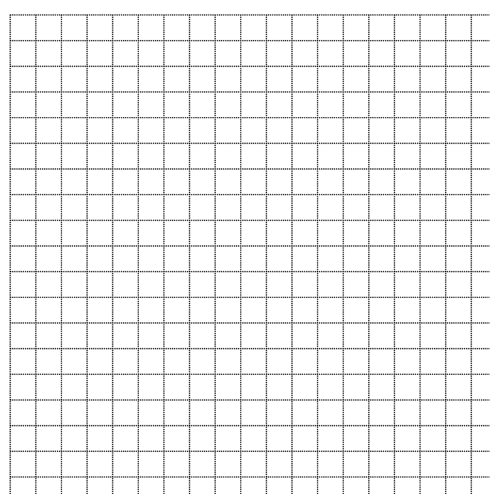
Graph 3:



Graph 4:

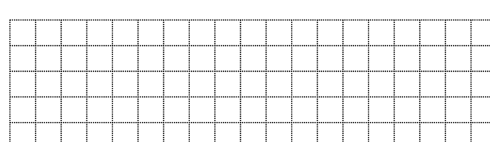
Page 384 – Example 1

Use transformations to sketch: $y=\log_3(x+9)+2$



Assignment: Page 389 #1, 2

Examples



1. Use transformations to sketch the graph of $y = \log(x - 10) - 1$.

b) Identify the following characteristics of the graph of the function.

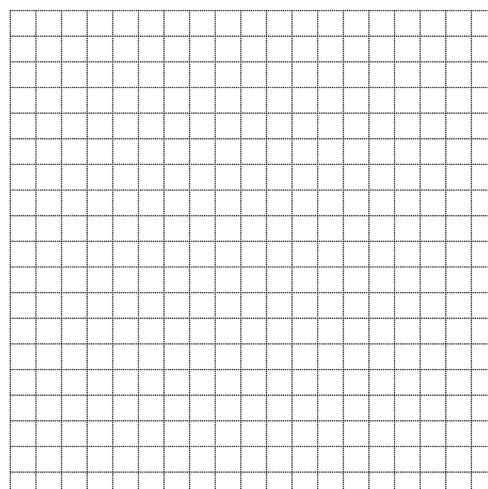
i) the equation of the asymptote

ii) the domain and the range

iii) the *y*-intercept, if it exists

iv) the *x*-intercept, if it exists

2. a) Determine the parameters and use mapping notation to sketch the graph of $y = 2\log_3(-x + 1)$.



b) Identify the following characteristics of the graph of the function.

i) the equation of the asymptote

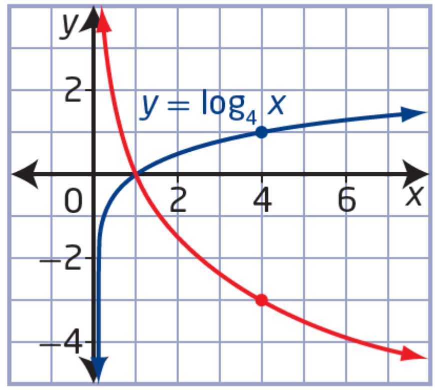
ii) the domain and the range

iii) the *y*-intercept, if it exists

iv) the *x*-intercept, if it exists

See Example 3 (page 387).

3. **Your Turn (Page388).** The graph at right was generated by ***stretching*** and ***reflecting*** the graph of $y = \log_4 x$. Write the equation of the second function on the graph.



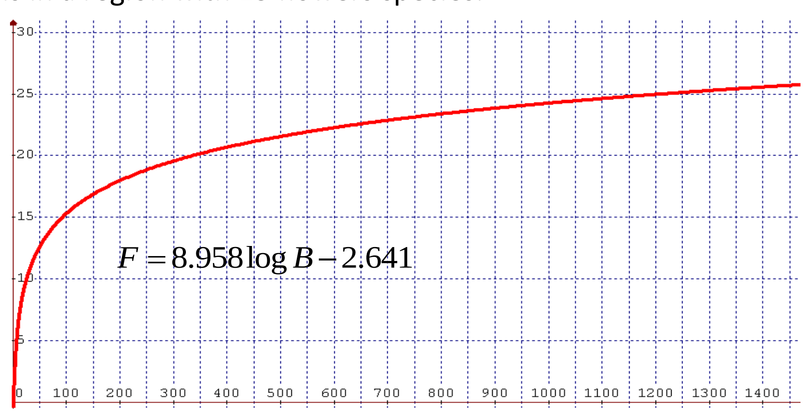
4. Only a horizontal translation (or ____________) has been applied to the graph of $y = \log_4 x$ so that the graph of the transformed image passes through the point (6, 2). Determine the equation of the transformed image.

<u>Application (Page 389 – Your Turn)</u>

5. There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, *F*, of flower species that a butterfly feeds on and the number of butterflies observed, *B*, can be modelled by the function $F = -2.641 + 8.958 \log B$.

   a) How many flower species would you expect to find if you observed 100 butterflies?

   b) Predict the number of butterfly observations in a region with 25 flowers species.



<u>Assignment</u>: Page 390 #4, 6, 8 – 10, 12, 13

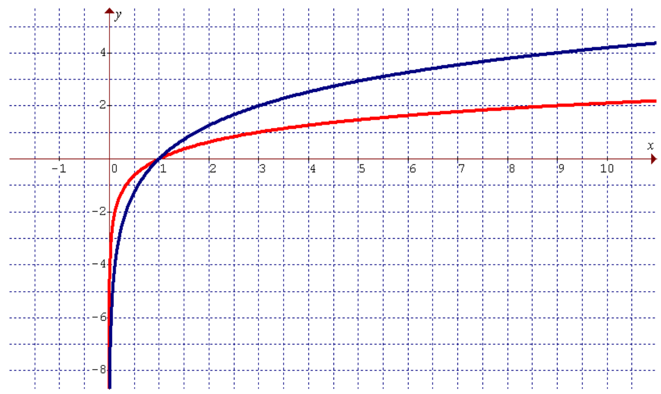
Pre-Calculus 30

8.2 Transformations of Logarithmic Functions (Day 1)

Given the base function $y = \log_c x$, multiple transformations can be applied using the general transformation model $y = a\log_c\left(b\left(x-h\right)\right)+k$. The mapping notation for multiple transformations would be

$(x, y) \rightarrow ($________, ________$)$.

1. Given the base function $f(x) = \log_3 x$, what sort of transformation occurred to produce the second graph?



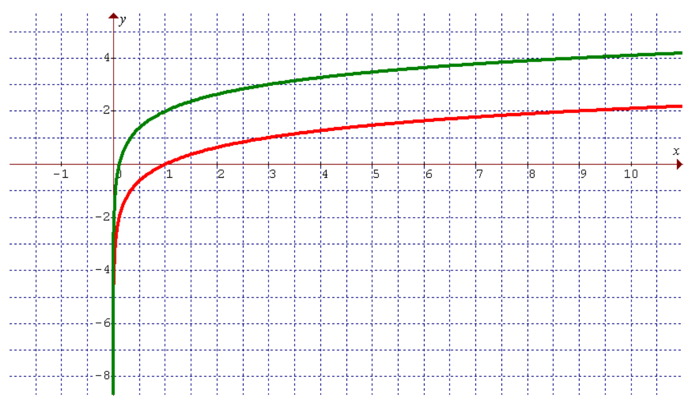
y=log$_3$ x

What characteristics changed as a result of this transformation?

What parameter produces this transformation? What is its value?

What is the equation of the second function?

2. Given the base function $f(x) = \log_3 x$, what sort of transformation occurred to produce the second graph?



y=log$_3$ x

What characteristics changed as a result of this transformation?

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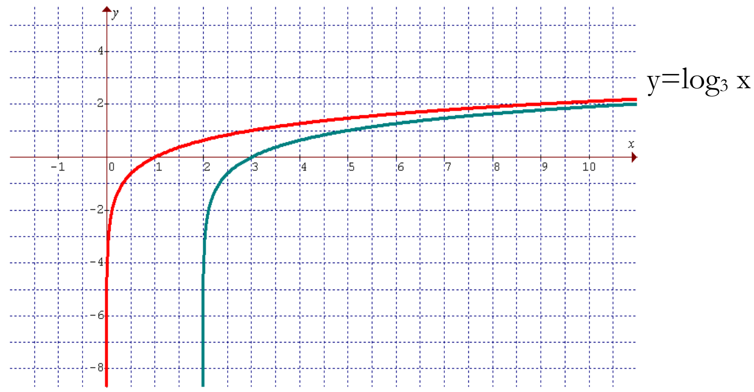
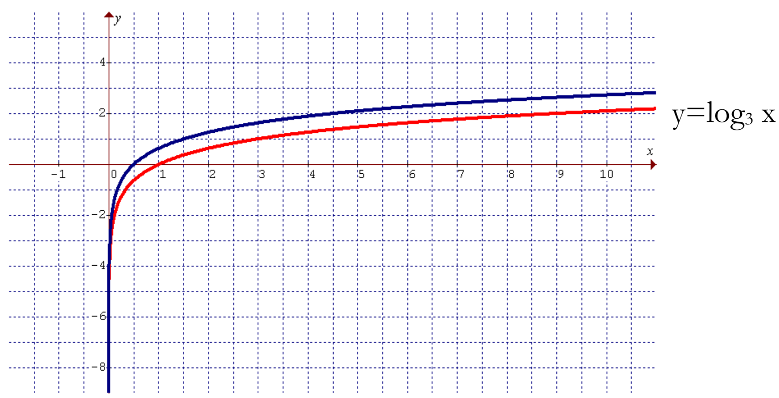


What is the equation of the second function?

4. Given the base function $f(x)=\log_3 x$, what sort of transformation occurred to produce the second graph?

What characteristics changed as a result of this transformation?

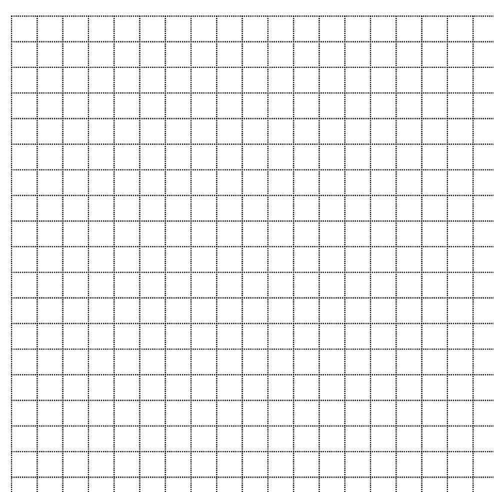
What parameter produces this transformation? What is its value?



What is the equation of the second function?

Page 384 – Example 1

Use transformations to sketch: $y=\log_3(x+9)+2$



<u>Assignment</u>: Page 389 #1, 2

8.2 Transformations of Logarithmic Functions (Day 2)

Examples

1. Use transformations to sketch the graph of $y = \log(x-10)-1$.

b) Identify the following characteristics of the graph of the function.

i) the equation of the asymptote

ii) the domain and the range

iii) the *y*-intercept, if it exists

iv) the *x*-intercept, if it exists

2. a) Determine the parameters and use mapping notation to sketch the graph of $y = 2\log_3(-x+1)$.

b) Identify the following characteristics of the graph of the function.

i) the equation of the asymptote

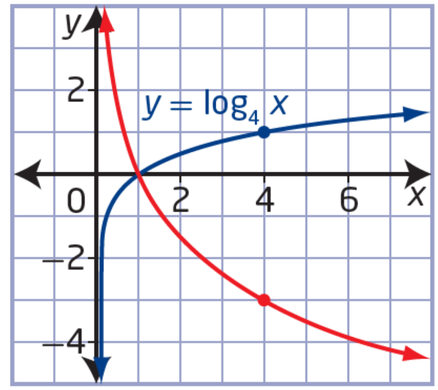
ii) the domain and the range

iii) the *y*-intercept, if it exists

iv) the *x*-intercept, if it exists

See Example 3 (page 387). Determining the equation of a function, given its graph.

3. **Your Turn (Page388).** The graph at right was generated by ***stretching*** and ***reflecting*** the graph of $y = \log_4 x$. Write the equation of the second function on the graph.



4. Only a horizontal translation (or ____________) has been applied to the graph of $y = \log_4 x$ so that the graph of the transformed image passes through the point (6, 2). Determine the equation of the transformed image.

<u>**Application (Page 389 – Your Turn)**</u>

5. There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, *F*, of flower species that a butterfly feeds on and the number of butterflies observed, *B*, can be modelled by the function $F = -2.641 + 8.958 \log B$.

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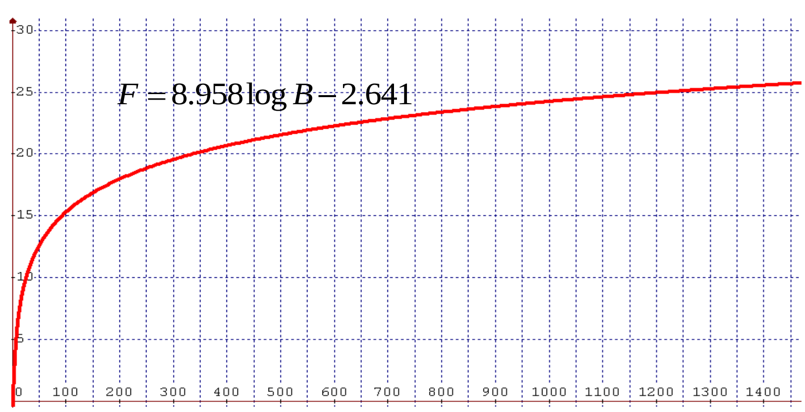
   b) Predict the number of butterfly observations in a region with 25 flowers species.



<u>**Assignment**</u>**:** Page 390 #4, 6, 8 – 10, 12, 13

8.2 – Transformations of Logarithmic Functions

In section 8.1, you were introduced to exponential functions which are written in the form $y = \log_c x$, where $c > 1$ and $c \neq 1$.

- What are the characteristics of the base function when $c > 1$?

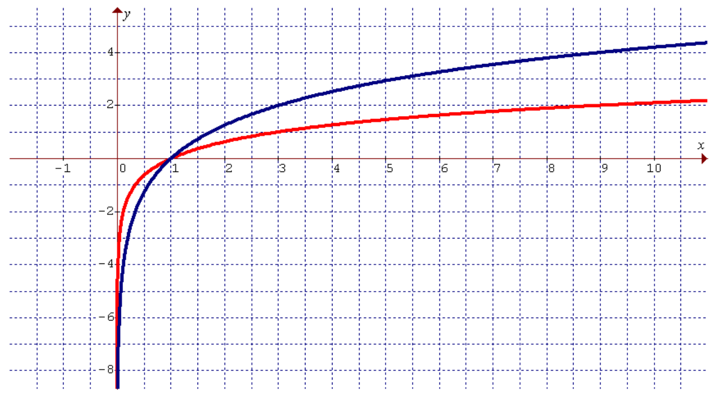
- What are the characteristics of the base function when $0 < c < 1$?

- Is there a relationship between the graphs of the first group and the second group? Why does this happen?

Investigate:

In every chapter, we have studied specific base functions then applied to three types of transformations: ___________________, ___________________, and ________________. In this lesson, we will apply these transformations to the new base function, $y = \log_c x$.

1. Given the base function $f(x) = \log_3 x$, what sort of transformation occurred to produce the second graph?

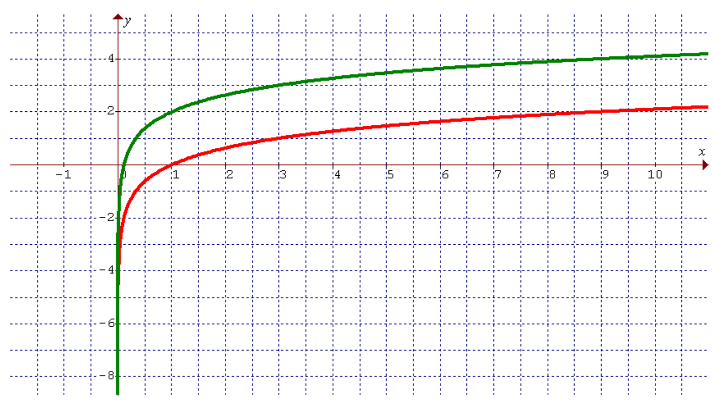


What characteristics changed as a result of this transformation?

What parameter produces this transformation? What is its value?

What is the equation of the second function?

2. Given the base function $f(x) = \log_3 x$, what sort of transformation occurred to produce the second graph?



What characteristics changed as a result of this transformation?

What parameter produces this transformation? What is its value?

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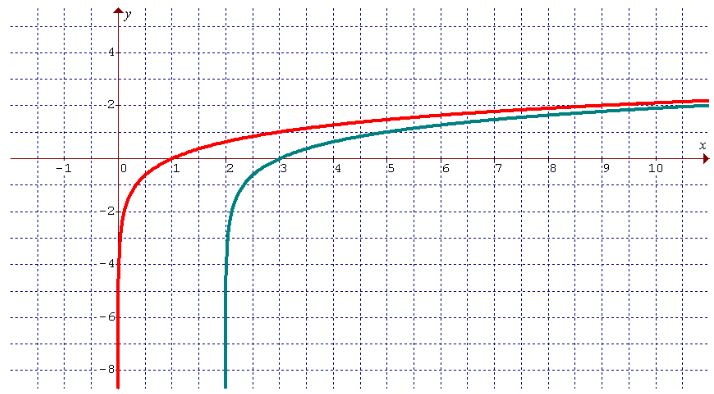
3. Given the base function $f(x) = \log_3 x$, what sort of transformation occurred to produce the second graph?

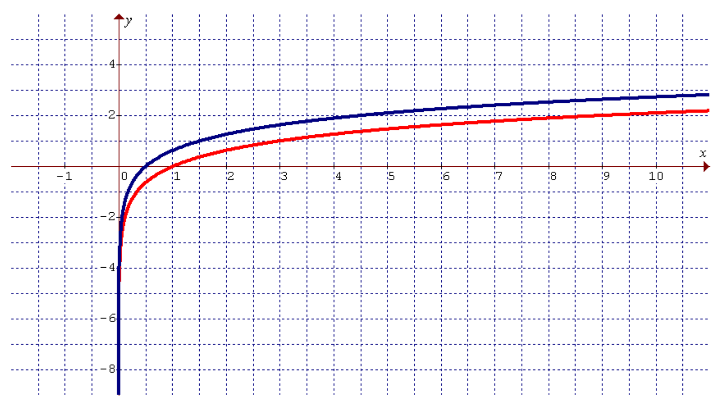


What characteristics changed as a result of this transformation?

What parameter produces this transformation? What is its value?

What is the equation of the second function?

4. Given the base function $f(x) = \log_3 x$, what sort of transformation occurred t o produce the second graph?



What characteristics changed as a result of this transformation?

What parameter produces this transformation? What is its value?

What is the equation of the second function?

Reminder: Given base function or parent function $y = \log_c x$, multiple transformations can be applied using the general transformation model $y = a\log_c\left(b\left(x-h\right)\right)+k$. The mapping notation for multiple transformations would be: $(x, y) \rightarrow ($________, ________$)$.

Examples:

1a) Use transformations to sketch the graph of $y = \log\left(x-10\right)-1$.

b) Identify the following characteristics of the graph of the function.

i) the equation of the asymptote

ii) the domain and the range

iii) the *y*-intercept, if it exists

iv) the *x*-intercept, if it exists

2a) Use transformations to sketch the graph of $y = 2\log_3(-x+1)$.

b) Identify the following characteristics of the graph of the function.

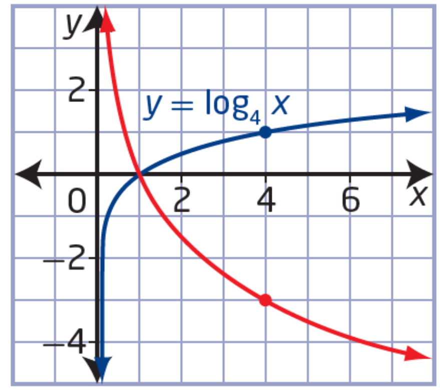
i) the equation of the asymptote

ii) the domain and the range

iii) the *y*-intercept, if it exists

iv) the *x*-intercept, if it exists

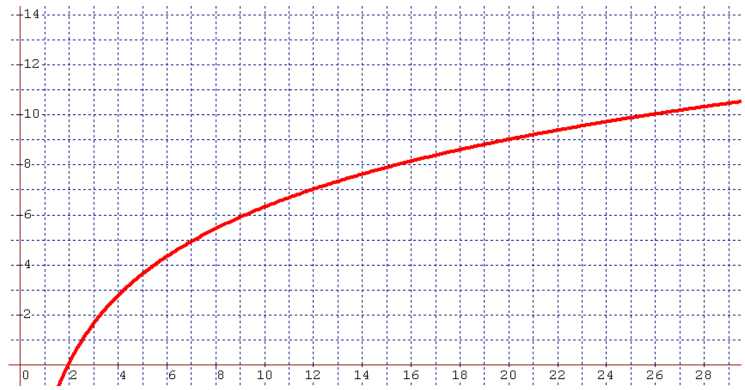
3. Given the base function $y = \log_4 4x$, write the equation of the second function on the graph.



4. Only a horizontal translation has been applied to the graph of $y = \log_4 x$ so that the graph of the transformed image passes through the point (6, 2). Determine the equation of the transformed image.

Application:

5. There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, *F*, of flower species that a butterfly feeds on and the number of butterflies observed, *B*, can be modelled by the function $F = -2.641 + 8.958 \log B$.



Predict the number of butterfly observations in a region with 25 flowers.

Pre-Calculus 30 **8.3 Laws of Logarithms (Day 1)**

Laws of Logarithms

1. Show that $\log(1000 \times 100) \neq (\log 1000)(\log 100)$. Find the value of both sides using a calculator.

Use a calculator to find the approximate value of each expression to four decimal places.

$\log 6 + \log 5 =$	$\log 30 =$
$\log 11 + \log 9 =$	$\log 99 =$
$\log 7 + \log 3 =$	$\log 21 =$

Product Law: $\log_c MN =$

Example: Write $\log 1000 + \log 100$ as a single logarithm.

2. Show that $\log\frac{1000}{100} \neq \frac{\log 1000}{\log 100}$. Find the value of both sides using a calculator.

Use a calculator to find the approximate value of each expression, to four decimal places.

$\log 48 - \log 4 =$	$\log 12 =$
$\log 72 - \log 2 =$	$\log 36 =$
$\log 35 - \log 5 =$	$\log 7 =$

Quotient Law: $\log_c \frac{M}{N} =$

Example: Write $\log 1000 - \log 100$ as a single logarithm.

3. Show that $\log 1000^2 \neq (\log 1000)^2$. Find the value of both sides using a calculator.

Use a calculator to find the approximate value of each expression, to four decimal places.

$3\log 5 =$	$\log 125 =$
$2\log 7 =$	$\log 49 =$
$4\log 2 =$	$\log 16 =$

<u>Power Law</u>: $\log_c M^P =$

<u>Example</u>: Write $2\log 1000$ as a logarithm without a coefficient.

<u>In summary</u>

Product Law : $\log_c MN = \log_c M + \log_c N$

Quotient Law : $\log_c \frac{M}{N} = \log_c M - \log_c N$

Power Law : $\log_c M^P = P\log_c M$

<u>Note</u>: These laws work for a logarithm of any base!

<u>Page 395 Example 1</u> – Write each expression in terms of individual logarithms of x, y, and z.

1. $\log_5 \frac{xy}{z}$

2. $\log_7 \sqrt[3]{x}$

3. $\log_6 \frac{1}{x^2}$

4. $\log \frac{x^3}{y\sqrt{z}}$

<u>Page 396 Example 2</u> – Use laws of logarithms to simplify and evaluate each expression.

1. $\log_6 8 + \log_6 9 - \log_6 2$

2. $\log_7 7\sqrt{7}$

3. $2\log_2 12 - \left(\log_2 6 + \frac{1}{3}\log_2 27\right)$

<u>Assignment:</u> **Handout #1 – 23, 28 – 41**

Pre-Calculus 30 **8.3 Laws of Logarithms (Day 2)**

Example 3 (Page 397) – Write as a single logarithm in simplest form. State ***all restrictions*** (logarithmic and rational restrictions) on the variable. Remember: bases must be positive and ≠ 1; can't take the log of a negative number.

1. $\log_7 x^2 + \log_7 x - \frac{5\log_7 x}{2}$

2. $\log_5 (2x-2) - \log_5 \left(x^2 + 2x - 3\right)$

Assignment: Page 400 #1 – 3, 8 – 12

8.4 Logarithmic and Exponential Equations

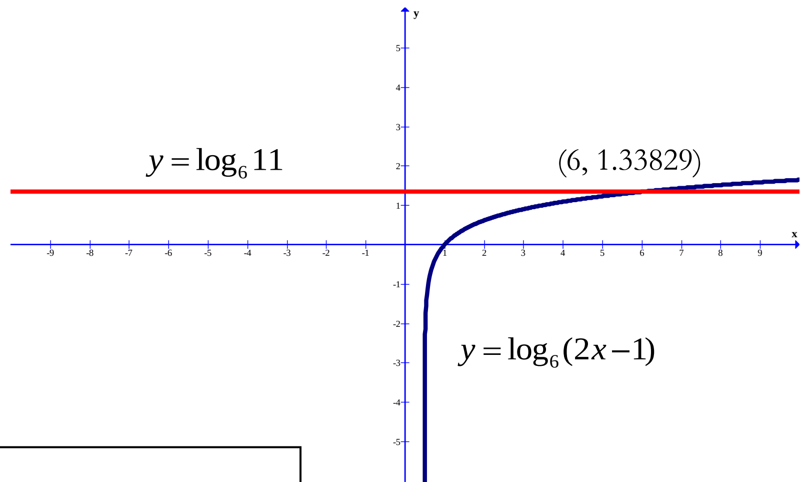
Logarithmic Equation – an equation containing the logarithm of a variable.

Logarithmic Equations can be solved ***graphically*** or ***algebraically.***

Example: Solve $\log_6(2x-1)=\log_6 11$ graphically.

Graph $y=\log_6(2x-1)$ and $y=\log_6 11$ on the same set of axes and find the x-coordinate of the point of intersection. (see graph below)

The point of intersection is (6, 1.33829) so ss = { 6 }



Given that *c, L, R* > 0 and c ≠1

- If $\log_c L=\log_c R,$ then $L=R$
- If $L=R,$ then $\log_c L=\log_c R$
- $\log_c L=R$ can be written as $L=c^R$
- substitute root into original equation to check if all logarithms are defined.

A. Solving Logarithmic Equations

- If each side contains the same log base, then the expressions are equal. (Write as single logarithms first)

a) $\log_6(2x-1)=\log_6 11$

b) $\log_7 x+\log_7 4=\log_7 12$

- If just one side contains a "log", change to exponential form and solve.

c) $\log_3(x^2-8x)^5=10$

- Combine and isolate the **logarithmic terms**. Apply appropriate law of logarithms.

d) $\log_2(x-6)=3-\log_2(x-4)$

B. Solving Exponential Equations Using Logarithms

- Take the common log of both sides and use **laws of logarithms** to isolate your variable; then solve with your calculator.

  a) $2^x = 2500$

  b) $8(3^{2x}) = 568$

  c) $5^{x-3} = 1700$

  d) $6^{3x+1} = 8^{x+3}$

Assignment: Page 412 #1, 2, 4, 5 (Day 1)
Page 412 #7ab, 8acd, 9, 11, 13, 15 (Day 2)