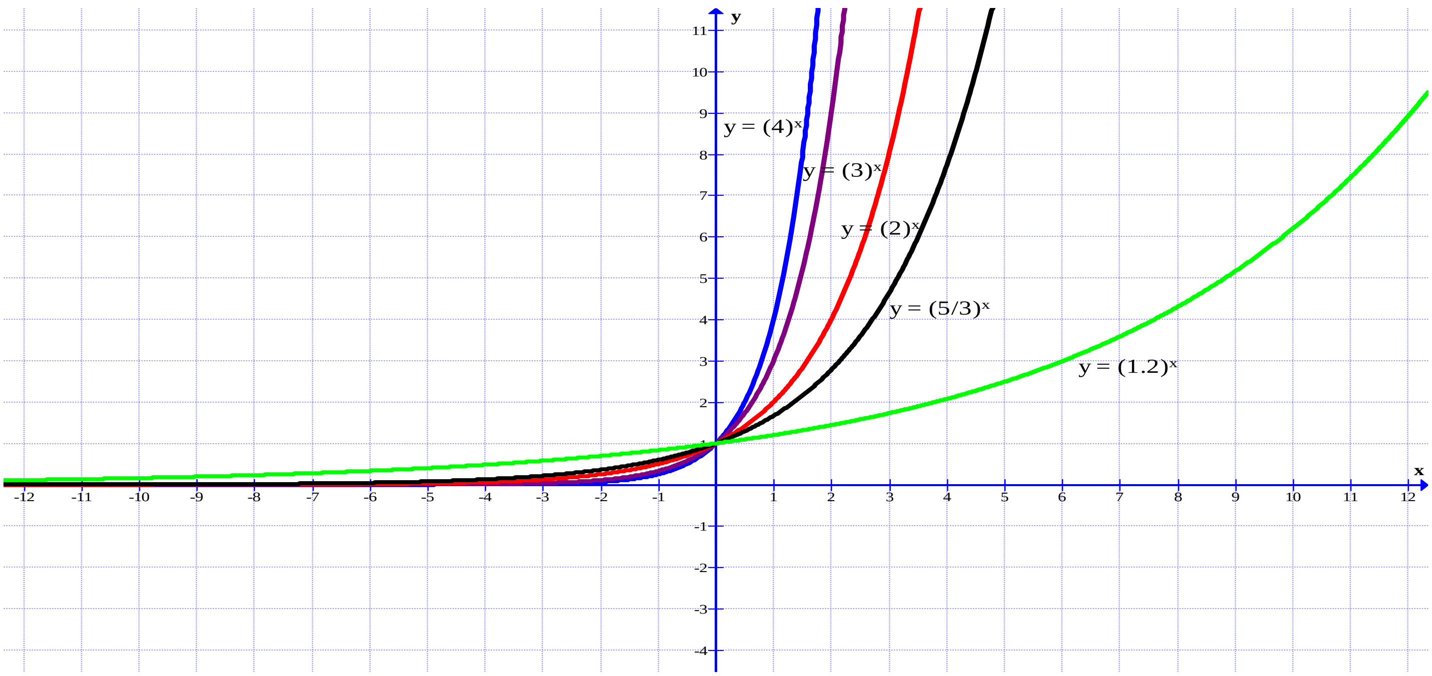
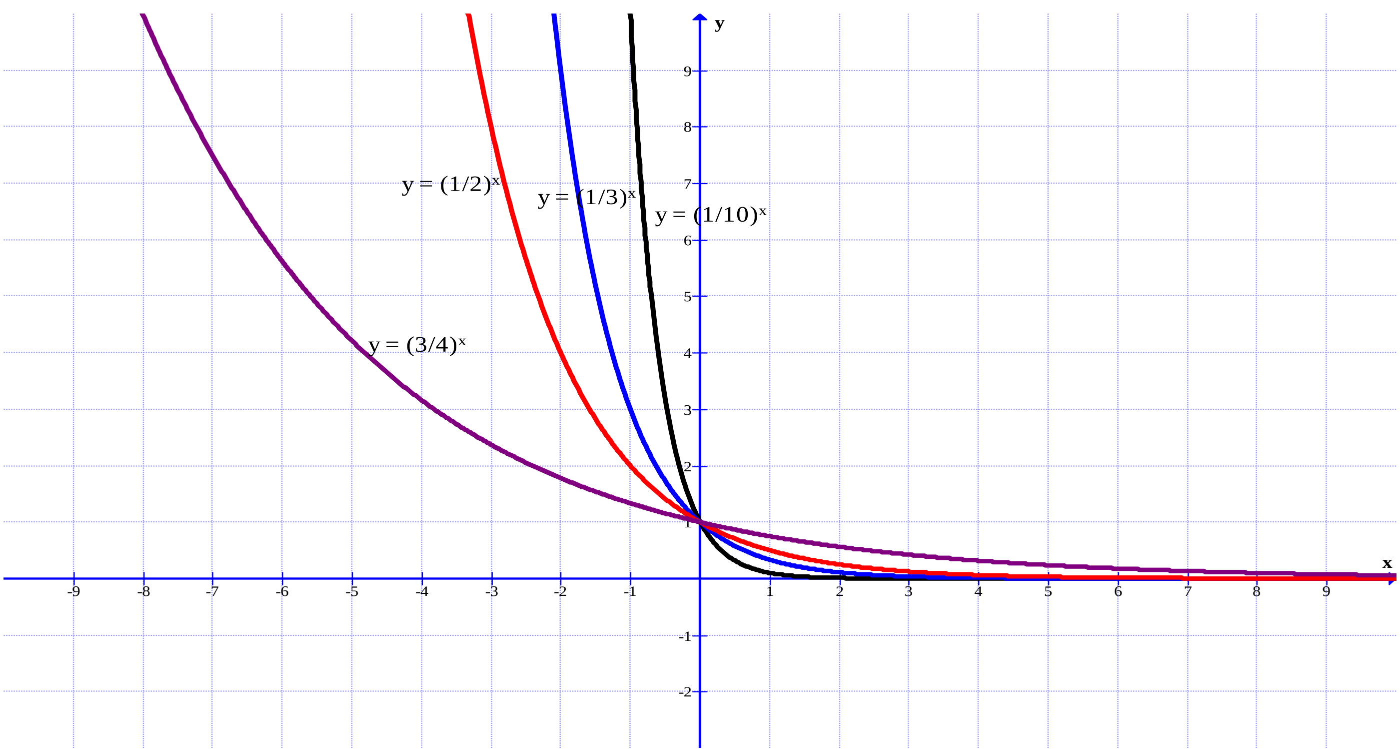
Pre-Calculus **7.1 Characteristics of Exponential Functions (Day 1)**

An **exponential function** is written in the form $y = c^x$, where $c > 0$.

- $y = c^x,\ c > 1$ What do you notice about these curves?



- $y = c^x,\ 0 < c < 1$ What do you notice about these curves?



Example 1 (page 336) See graphs on other side.

a) $y = 4^x$.

Domain:
Range:
x-intercept:
y-intercept:
increasing or decreasing?
Horizontal asymptote:

b) $y = \left(\frac{1}{2}\right)^x$.

Domain:
Range:
x-intercept:
y-intercept:
Increasing or decreasing?
Horizontal asymptote:

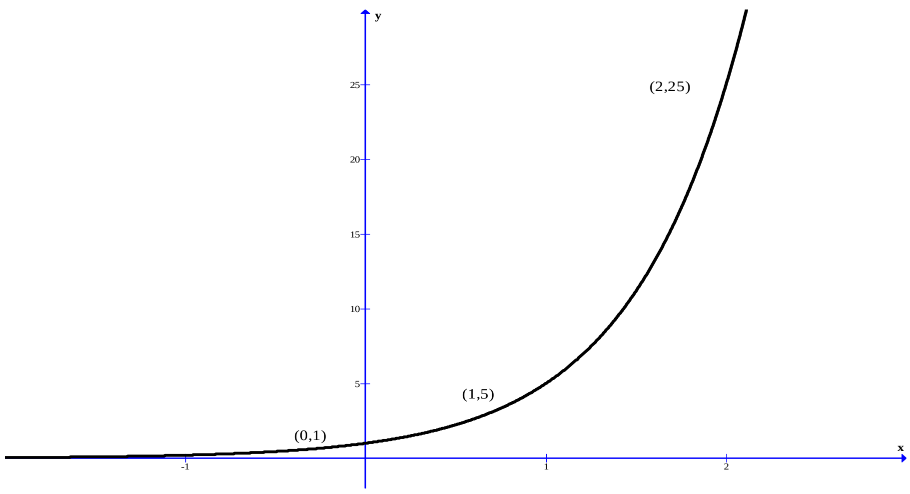
Summary: For the graph $y = c^x$

- **When c > 1 the graph of $y = c^x$ is ______________________**
- **When 0 < c < 1 the graph of $y = c^x$ is ______________________**
- **domain is**
- **range is**
- **y-intercept**
- **x-intercept**
- **horizontal asymptote**

Example 2 (page 338) Writing the function when given its graph.

- Write down key points (table of values or ordered pairs) that are easy to read on the curve
- Look for a pattern in the ordered pairs. Ask yourself "As *x* increases by 1, the value of *y* increases/decreases by what factor?" Choose a point other than (0, 1) to verify your decision.

Your Turn (Page 339) What function of the form $y = c^x$ can be used to describe the graph shown?



Assignment: Page 342 #1 – 5

Pre-Calculus **<u>7.1 Characteristics of Exponential Functions - Applications</u> (Day 2)**

- **Exponential growth** is an ***increasing*** pattern of values that can be modeled by a function of the form $y = c^x$, where $c > 1$. Can you think of an example of exponential growth?

- **Exponential decay** is a ***decreasing*** pattern of values that can be modeled by a function of the form $y = c^x$, where $0 < c < 1$. Can you think of an example of exponential decay?

- **Half-life** is the length of time for an unstable element to spontaneously decay to one half its original mass. Can you think of an example of half-life?

Page 340 – Example 2 A radioactive sample of radium (Ra-225) has a half-life of 15 days. The mass, *m*, in grams, of Ra-225 remaining over time, *t*, in 15-day intervals, can be modeled using the exponential graph shown on page 340

a) What is the initial mass of Ra-225 in the sample?

What value does the mass of Ra-225 remaining approach as time passes?

b) What is the domain of this function?

What is the range of this function?

c) Write the exponential decay model that relates the *mass* of Ra-225 remaining to *time*, in 15-day intervals. (Write the equation for the function)

d) Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass. (Use the graph on page 340)

Homework

1. The number of rabbits, y, in a certain population after t months is modeled over a short period of time by the function $y = 20(2)^t$.

a) Determine the initial number of rabbits, that is the number of rabbits when $t = 0$.

b) Determine the number of rabbits after 4 months.

c) Determine the number of rabbits after 2 years.

d) Why is this function not an appropriate model for the population growth over a **long** period of time?

e) Is this an example of exponential growth or exponential decay?

2. The value, y, of a car after t years is modeled by the function $y = 25\,000(0.85)^t$.

a) Determine the initial value of the car, that is the value when $t = 0$.

b) Determine the value of the car after 1 year.

c) Determine the value of the car after 5 years.

d) Determine the value of the car after 30 years.

e) What factors might cause this function to not be a good model for the value of the car over a long period of time?

f) Is this an example of exponential growth or exponential decay?

Answers

1. a) 20 b) 320 c) 335 544 320 d) Answers will vary – biology e) exponential growth
2. a) $25 000 b) $21 250 c) 11 092.63 d) $190.77 e) Collector's item f) exponential decay

Assignment: Page 343 #6 – 8

7.2 Transformations of Exponential Functions

In section 7.1, you were introduced to exponential functions which are written in the form $y = c^x$, where *c*, the base, is a real number, strictly positive, and not equal to 1.

- What are the characteristics of the base function when $c > 1$? Sketch an example.

- What are the characteristics of the base function when $0 < c < 1$? Sketch an example.

Investigate:

Transformations can alter the equation or graph of a function. Describe the general roles of the parameters *a, b, h* and *k.*

a:

b:

h:

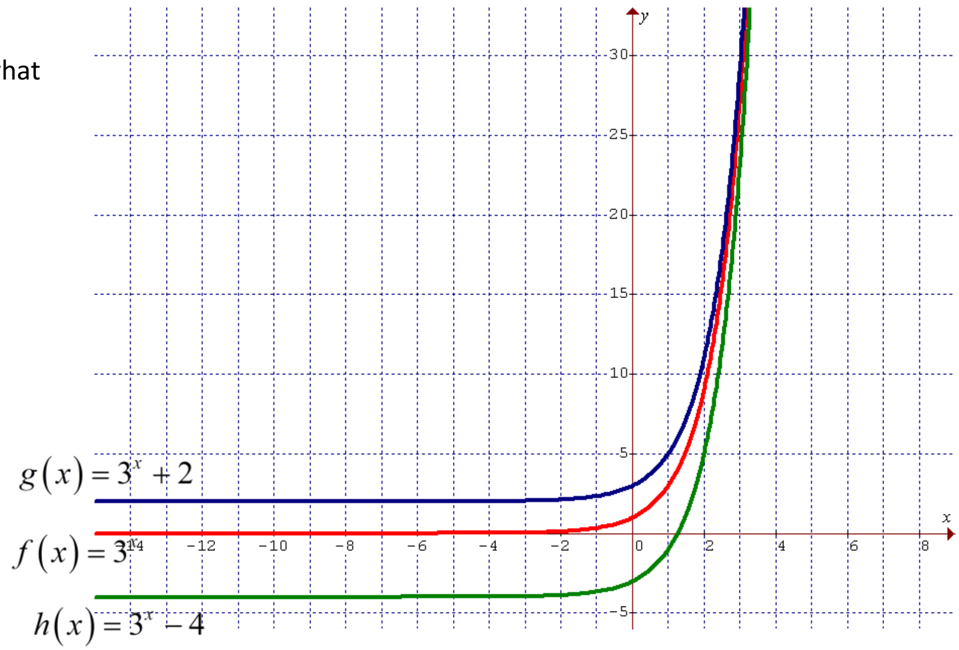
k:

Given the base function $y = c^x$, multiple transformations can be applied using the general transformation model $y = a(c)^{b(x-h)} + k$.

The mapping notation for multiple transformations would be: $(x, y) \rightarrow ($____________, ____________$)$

Consider the graphs of the following sets of functions:

Given the base function $f(x) = 3^x$, what sort of transformation occurred to produce the other two graphs?



What is the parameter?

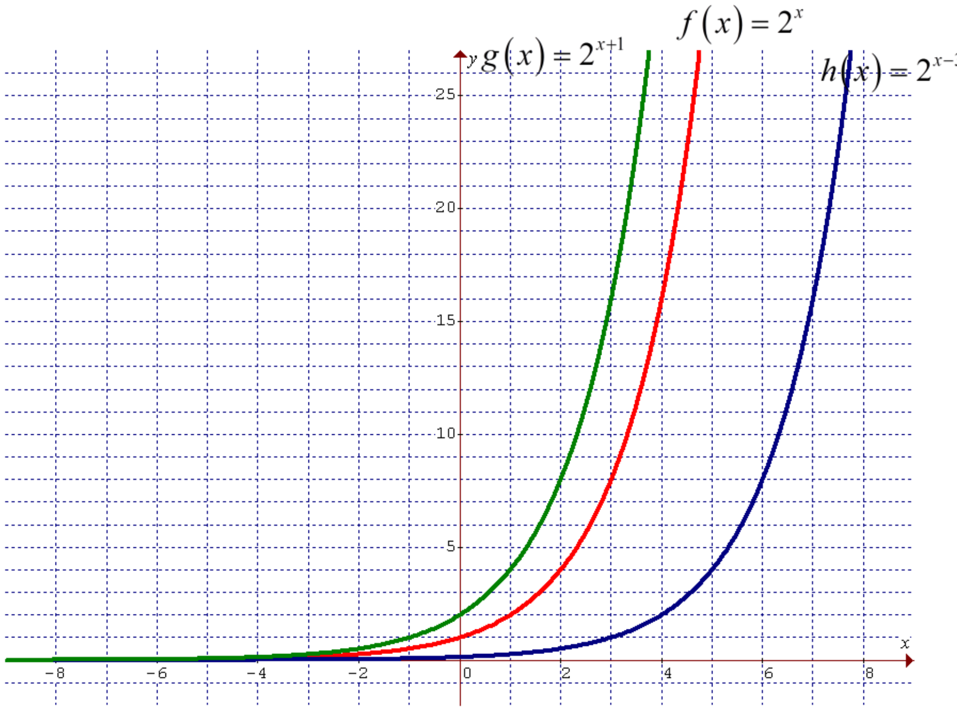
What is the value of this parameter in:

$g(x) = 3^x + 2$?

$h(x) = 3^x - 4$?

On the graph, label the y-intercepts.

Given the base function $f(x)=2^x$, what sort of transformation occurred to produce the other two graphs?



What is the parameter?

What is the value of this parameter in:

$g(x)=2^{x+1}$?

$h(x)=2^{x-3}$?

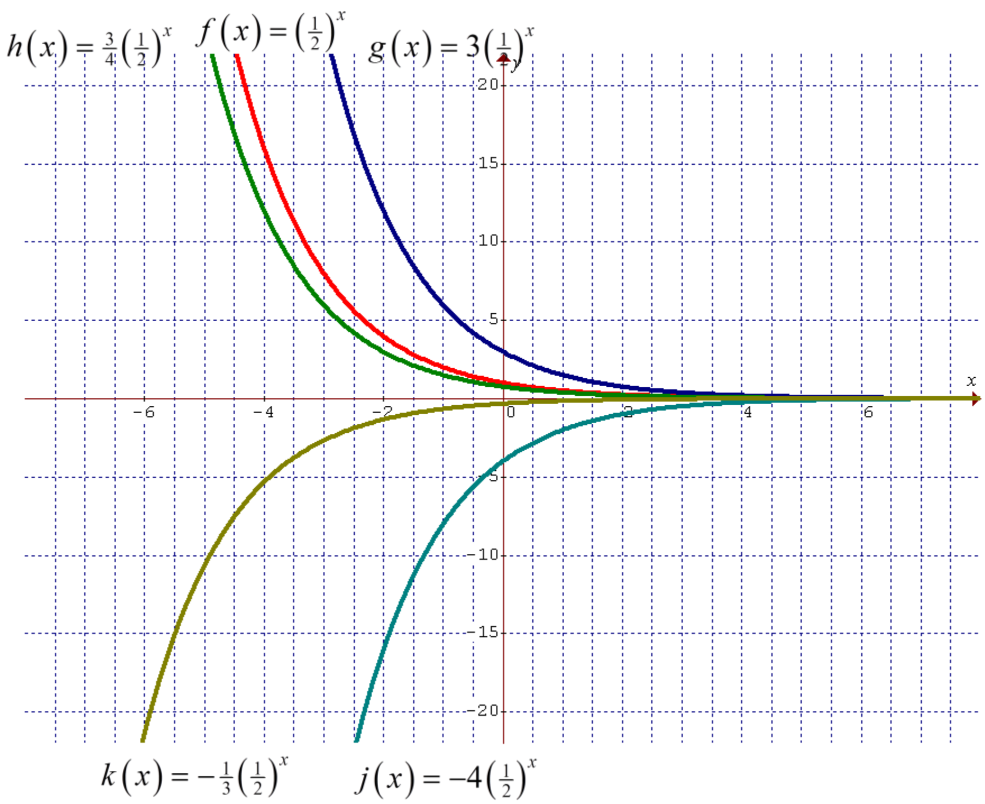
On the graph, label the

y-intercept of $f(x)=2^x$

Where has this point moved on the other two graphs? Label these points on the graph.

Describe the roles of the parameters *h* and *k* in the functions of the form $y=a(c)^{b(x-h)}+k$.

Given the base function $f(x) = \left(\frac{1}{2}\right)^x$, what sort of transformation occurred to produce the other two graphs?



What is the parameter?

What is the value of this parameter in:

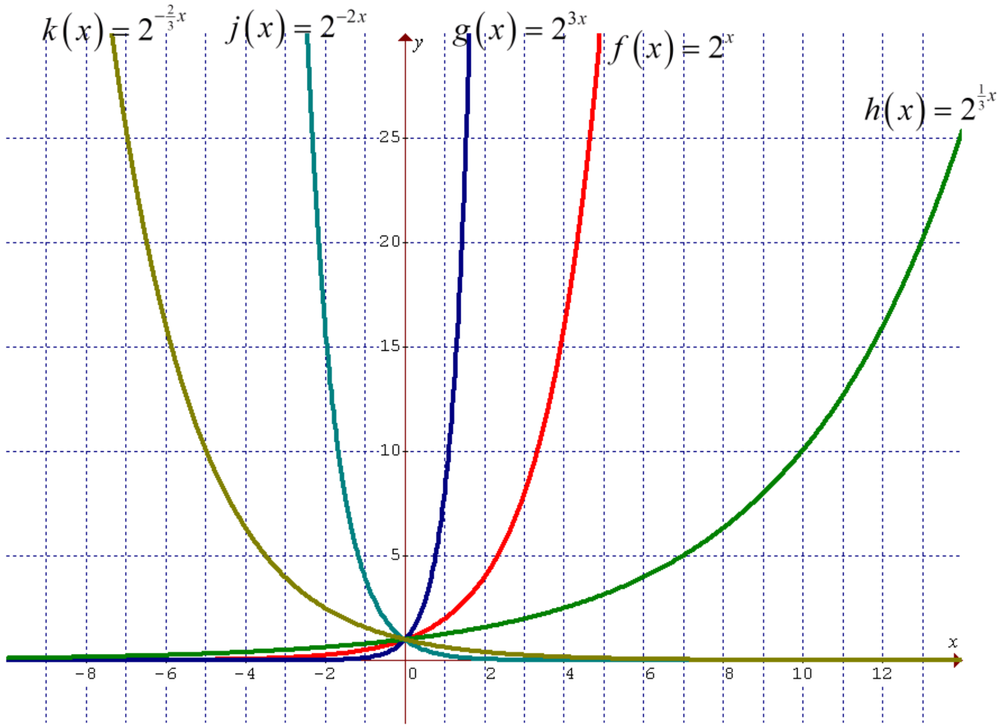
$h(x) = \frac{3}{4}\left(\frac{1}{2}\right)^x$?

$g(x) = 3\left(\frac{1}{2}\right)^x$?

$k(x) = -\frac{1}{3}\left(\frac{1}{2}\right)^x$?

$j(x) = -4\left(\frac{1}{2}\right)^x$?

Given the base function $f(x) = 2^x$, what sort of transformation occurred to produce the other two graphs?



What is the parameter?

What is the value of this parameter in:

$k(x) = 2^{-\frac{2}{3}x}$?

$j(x) = 2^{-2x}$?

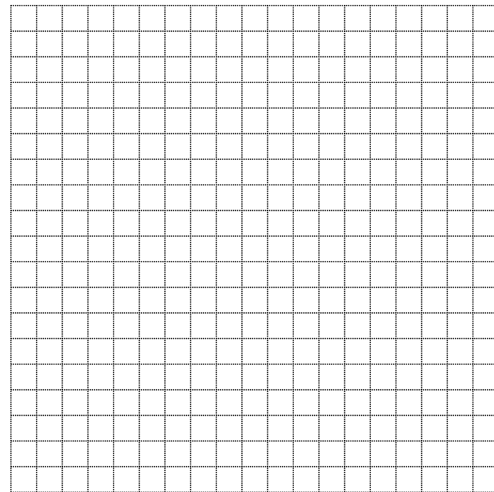
$g(x) = 2^{3x}$?

$h(x) = 2^{\frac{1}{3}x}$?

Describe the roles of the parameters *a* and *b* in the functions of the form $y = a(c)^{b(x-h)} + k$.

Examples:

1. Transform the graph of $y = 4^x$ to sketch the graph of $y = 4^{-2(x+5)} - 3$. Describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts.



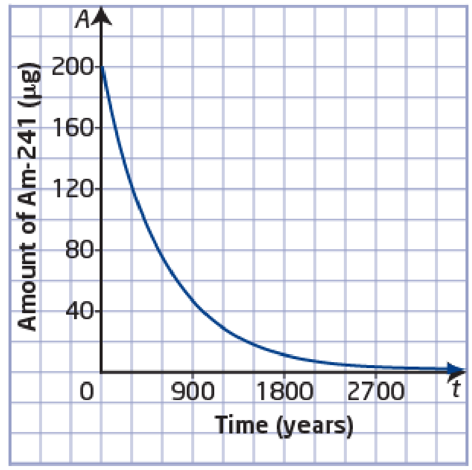
2. ***Describe*** how each parameter in exponential function $y = 0.5(3)^{-2(x+4)} + 7$ transforms the graph of the original function, $y = 3^x$. Do not sketch the graph.

See Example 2 (page 352)

Application:

3. The radioactive element americium (Am) is used in household smoke detectors. Am-241 has a half-life of approximately 432 years. The average smoke detector contains 200 µg of Am-241.

a) What is the transformed exponential function that models the graph showing the radioactive decay of 200 µg of Am-241?



b) Identify how each of the parameters of the function relates to the transformed graph.

Assignment: Page 354 #1 – 7, 9, 11, 12

7.3 Solving Exponential Equations

Exponential equations – an equation that has a variable in the exponent

Substitute the value of *n* in to each exponential expression. Then rewrite each expression as an equivalent expression with base 2

n	$\left(\frac{1}{2}\right)^n$	2^n	4^n
-2			
-1			
0			
1			
2			

Example – Rewrite the following with a base of 2

a) 32

b) 16^3

c) $(1/64)^{1/3}$

Example – Rewrite the following with a base of 3

a) 27^5

b) $\sqrt[3]{243}$

c) $\left(\frac{\sqrt{3}}{81}\right)^{-3}$

It is often helpful to rewrite exponential expressions using a different base since:

If $\boldsymbol{c^x = c^y}$ then $\boldsymbol{x = y}$ (for $c \neq -1, 0, 1$)

<u>Example</u>: Solve

a) $4^{2x} = 8^{x+1}$

b) $64^{4x} = 16^{x+5}$

c) $9^{x-7} = 27^{2x-9}$

d) $8^{x-2} = (1/4)^{x+3}$

<u>Compound Interest</u>:

$A = P(1+i)^n$

A = Amount of money at the end of the investment
P = Principal amount deposited (or invested)
i = interest rate *per compounding period* (as a decimal)
n = number of compounding periods

Example - Page 365 #13 ab

<u>Assignment</u>: Solve the following exponential equations below

Page 364 #2, 4, 5, 11ab, 12ab, 14

a) $2^x = 32$

b) $4^{x-2} = 8^4$

c) $2^{x-5} = 4$

d) $4^{1-x^2} = 8^x$

e) $2^{x^2} = \left(16^{x-1}\right)\left(2^x\right)$

f) $4^{x-1} = \left(\frac{1}{2}\right)^{4x-1}$

g) $9^{2x} = \sqrt{27}$

h) $13^{x^2-4} = 1$

<u>Answers</u>: a) {5} b) {8} c) {7} d) {½, -2} e) {1, 4} f) {½} g) {3/8} h) {2, -2}