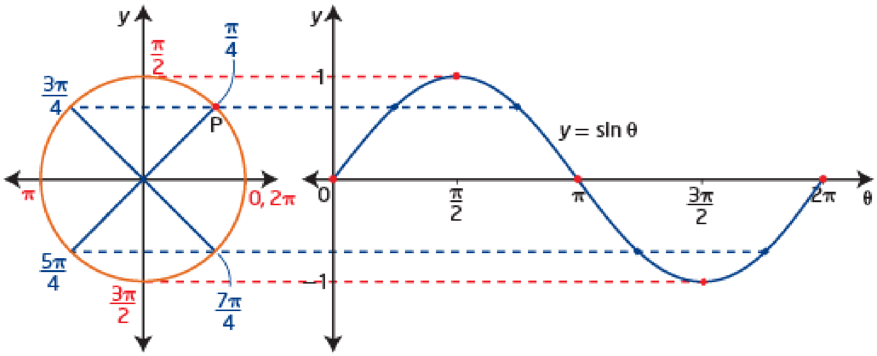
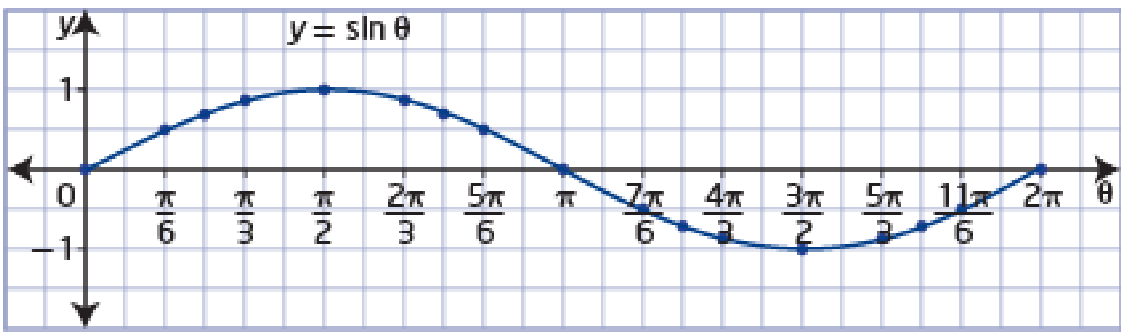
A. The Sine Function

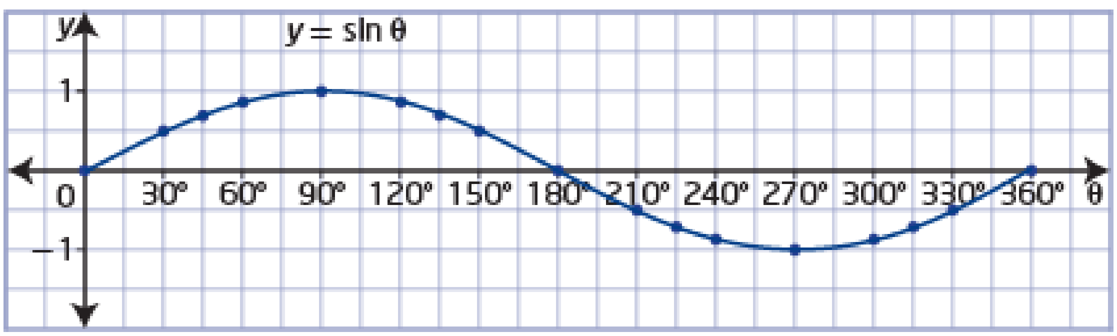
- The values of $\sin\theta$ can be transferred to a new view as seen below:



25° 240° 270° 300° 315° 330° 360°

θ Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0



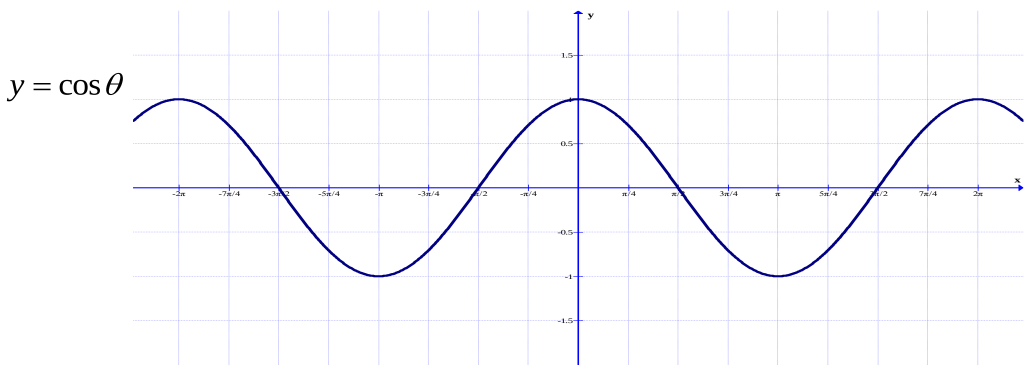


- The graph/curve is **periodic** and **continuous**.
- The **domain** is
- The **range** is
- The **maximum** value is
- The **minimum** value is
- The **amplitude** (maximum vertical distance above or below the horizontal centre) is
- The **period** (the horizontal length of one cycle) is
- The ***y*-intercept** is

B. The Cosine Function

Complete the table of values for cos θ. Then label the graph of cos θ below.

degrees	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Cos θ								



How is the graph of $y = \cos\theta$ similar to that of $y = \sin\theta$?

How is it different?

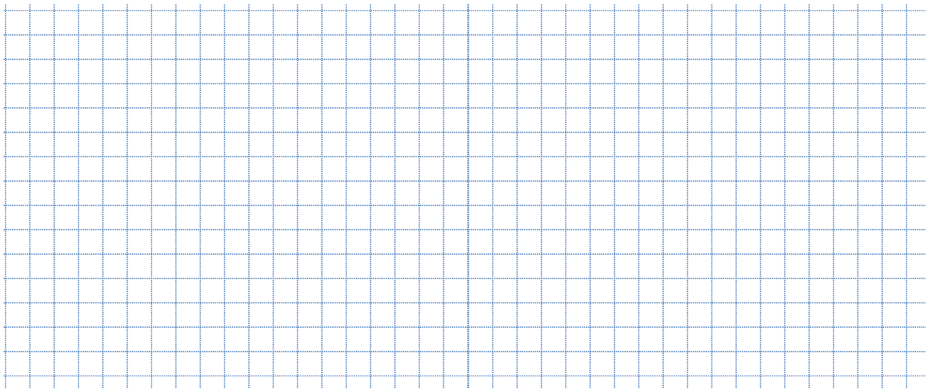
From the graph of the cosine function we can note the following observations:

- The graph is **periodic** and **continuous**.
- The **domain** is
- The **range** is
- The **maximum** value is
- The **minimum** value is
- The **amplitude** of the curve is
- The **period** is
- The ***y*-intercept** is

C. Determining the Amplitude of a Sine or Cosine Function

We know that any function in the form $y = af\left(x\right)$ is related to $y = f\left(x\right)$ by a vertical stretch by a factor of $|a|$. Remember that if $a < 0$, then the function must also be reflected in the *x*-axis.

Example: On the same set of axes, graph $y = \sin x$, $y = 2\sin x$, $y = 0.5\sin x$ and $y = -2\sin x$ for $0 \le x \le 2\pi$.



Determine the ***amplitude*** of each function by using the formula:

$$\boxed{\text{amplitude} = \frac{\text{max value} - \text{min value}}{2} \quad \text{or} \quad A = \frac{M - m}{2}}$$

a) $y = \sin x$

b) $y = 2\sin x$

c) $y = 0.5\sin x$

d) $y = -2\sin x$

Do the ***period*** lengths vary?

Determine the ***domain*** and ***range*** for each

For $y = \sin x$,

For $y = 2\sin x$,

For $y = 0.5\sin x$,

For $y = -2\sin x$,

Assignment: Page 233 #4, 7, 11

5.1 Graphing Sine and Cosine Functions (Day 2)

A. The Period of Sine and Cosine Functions

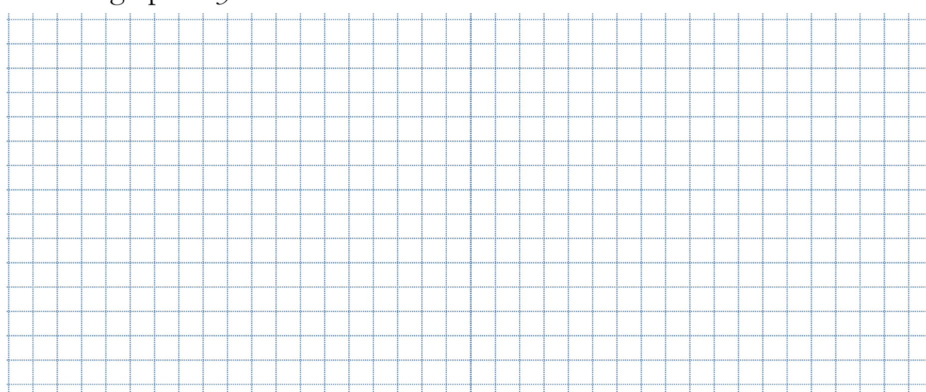
- Any function y = f(bx) is related to y = f(x) by a ***horizontal stretch*** by a factor of $\frac{1}{|b|}$ about the y-axis.
- The period of the base function of $y = \sin x$ or $y = \cos x$ is 2π.
- To determine the period of either of these functions when b $\neq 1$, solve the following compound inequality:

$0 \le x \le 2\pi$	Begin with the interval of one cycle of $y = \sin bx$ or $y = \cos bx$
$0 \le \lvert bx \rvert \le 2\pi$	Replace x with $\lvert b \rvert x$
$0 \le x \le \frac{2\pi}{\lvert b \rvert}$	Divide by $\lvert b \rvert$

Solving this inequality determines the length of a cycle for the sinusoidal curve (sin or cos), where the **start** of a cycle is at 0 and the **end** is at $\frac{2\pi}{|b|}$.

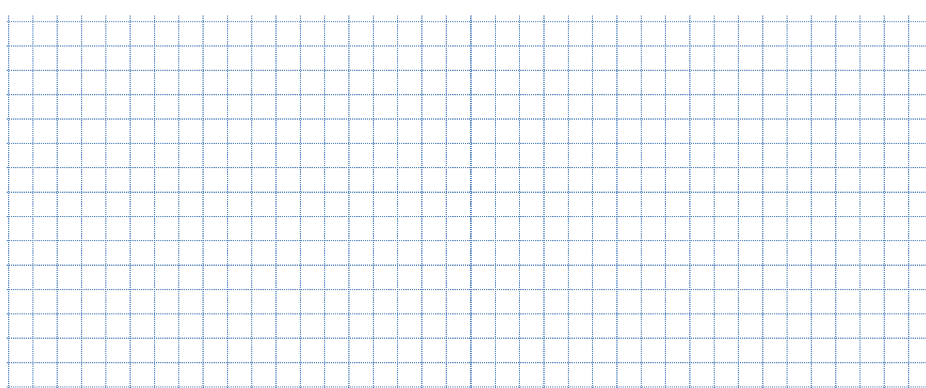
> The **period** for $y = \sin bx$ **or** $y = \cos bx$ is $\frac{2\pi}{|b|}$ (radians) or $\frac{360^\circ}{|b|}$ (degrees)

Example: Sketch the graph of the function $y = \sin 4x$ for $0 \le x \le 2\pi$. Determine the period of $y = \sin 4x$. (Divide each cycle into 4 equal parts). Sketch the graph of $y = \sin x$.



Example: Sketch the graph of the function $y = \cos\frac{1}{3}x$ for $0 \le x \le 6\pi$. Determine the period of $y = \cos\frac{1}{3}x$

(Divide each cycle into 4 equal parts).

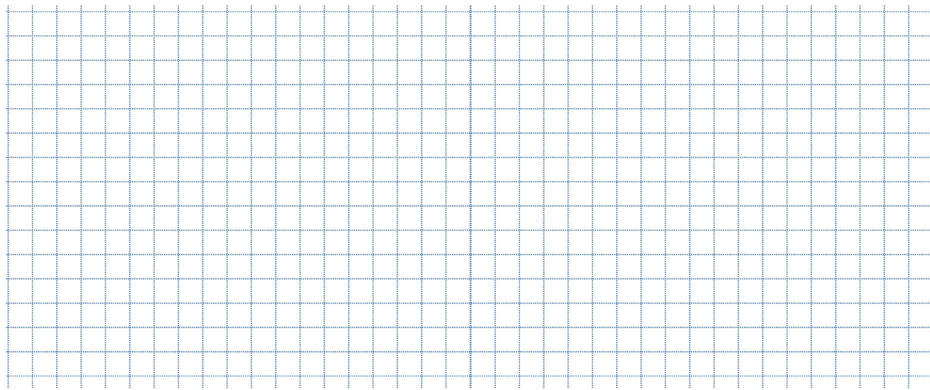


B. Sketching the Graphs of y = a *sin* bx and y = a *cos* bx

Example: Graph $y = 3\sin 2x$, showing at least 2 cycles.

$a =$ Therefore, the maximum value is _____ and the minimum value is _______.

$b =$ Therefore, the period is ________ . One cycle will start at $x =$ _______and end at $x =$ _______.
(Divide each cycle into 4 equal segments.)



amplitude:

***x*-intercepts:**

***y*-intercept:**

Domain:

Range:

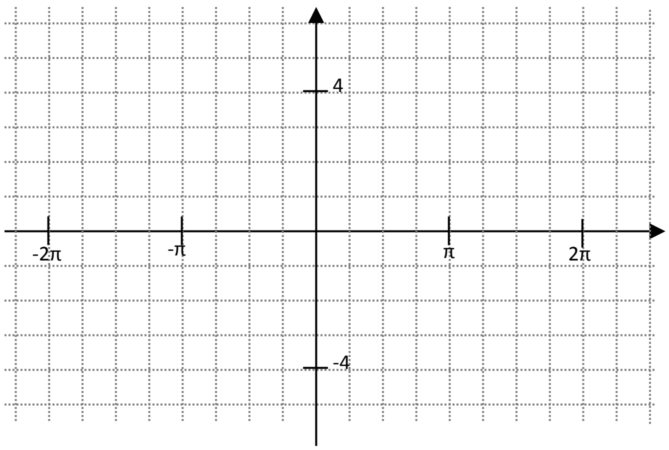
Assignment: Page 233 #5, 6, 8 – 10, 14, 15

Pre-Calculus 30

5.2 Transformations of Sinusoidal Functions (Day 1)

A. Graph of $y = \sin x + d$ and $y = \cos x + d$

1. Sketch the graph of $y = \sin x$. Identify the period length, the amplitude, the domain and the range.

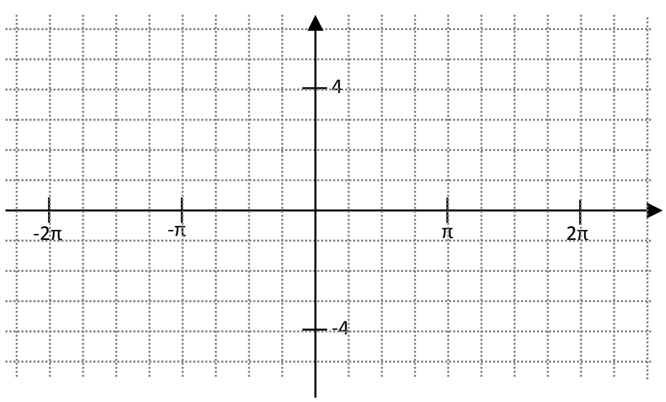


	$y = \sin x$	$y = \sin x + 1$	$y = \sin x - 2$
Period			
Amplitude			
Domain			
Range			

With your knowledge of transformations, what do you predict is the effect of *d*? What parameter was used in the other chapters to express this?

- ➢ Now graph $y = \sin x + 1$ and $y = \sin x - 2$ on the set of axes above.

2. Sketch the graph of $y = 2\cos x$ on the axes below. Identify the period length, the amplitude, the domain and the range.



	$y = 2\cos x$	$y = 2\cos x - 3$
Period		
Amplitude		
Domain		
Range		

- ➢ Now graph of $y = 2\cos x - 3$ on the set of axes above.

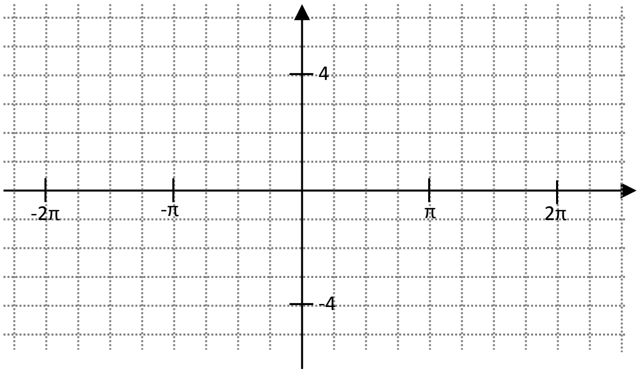
Vertical displacement is the vertical translation of the graph of a periodic function. It is denoted by *d*.

- Up if $d > 0$
- Down if $d < 0$

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

B. Graph of $y = sin(x - c)$ and $y = cos(x - c)$

3. Sketch the graph of $y = -3\sin x$ on the axes. Identify the period length, the amplitude, the domain and the range.

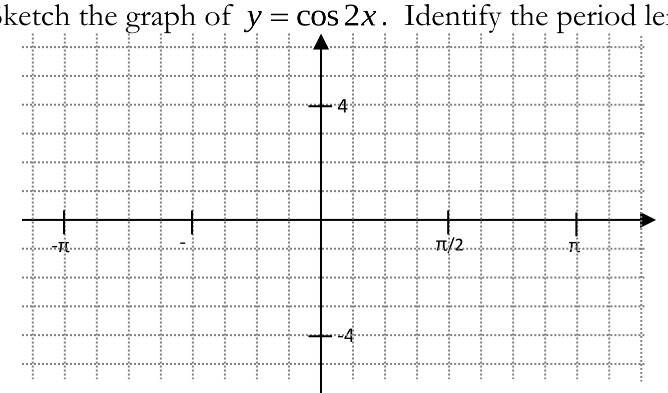


	$y = -3\sin x$	$y = -3\sin\left(x - \frac{\pi}{2}\right)$
Period		
Amplitude		
Domain		
Range		

With your knowledge of transformations, what do you predict is the effect of c? What parameter was used in the other chapters to express this?

- Now graph $y = -3\sin\left(x - \frac{\pi}{2}\right)$ on the set of axes above.

4. Sketch the graph of $y = \cos 2x$. Identify the period length, the amplitude, the domain and the range.



	$y = \cos 2x$	$y = \cos 2\left(x + \frac{\pi}{4}\right)$
Period		
Amplitude		
Domain		
Range		

- Now graph $y = \cos 2\left(x + \frac{\pi}{4}\right)$ on the set of axes above.

Use the language of transformations to compare the graphs of $y = \cos 2x$ and $y = \cos 2\left(x + \frac{\pi}{4}\right)$.

Phase shift is the horizontal translation of the graph of a periodic function. It is denoted by c.

- Right if $c > 0$
- Left if $c < 0$

sinusoidal functions	**amplitude**	**period length**	**phase shift (horizontal)**	**vertical displacement**
$y = a\sin b(x - c) + d$ $y = a\cos b(x - c) + d$				

Assignment: Page 250 #1 (no sketch), 2 (no sketch), 4 - 7

Pre-Calculus 30 **5.2 Transformations of Sinusoidal Functions (Day 2)**

Example: Identify period length, amplitude, phase shift and vertical displacement. $\boxed{y = a\sin b(x-c)+d}$

1. $y = 2\cos\frac{2}{3}\left(x-\frac{3\pi}{4}\right)+1$

amplitude:____________

period length:____________

phase shift:____________

vertical displacement:____________

2. $y = -10\sin\left(3x+\pi\right)-2$

amplitude:____________

period length:____________

phase shift:____________

vertical displacement:____________

Graphing – graph amplitude and period, then shift vertically and horizontally.

Example: Sketch the following functions over **two cycles**. Show quarter interval markings.
(Hint: Graph amplitude and period, then shift vertically and horizontally.)

1. $y = 5\cos\left(x-\frac{\pi}{2}\right)+1$

2. $y = -\sin 2\left(x+\frac{\pi}{3}\right)-3$

3. $y = -2\cos\left(2x+90^\circ\right)+1$

Assignment: Page 250 #1ef (sketch), 2ace(sketch), 3, 8, 10a, 11, 12ac, 14ab (i-v)

Interpreting Graphs of Sinusoidal Functions (Page 247 Example 5)

Prince Rupert, BC, has the deepest natural harbor in North America. The depth, d, in meters, of the berths for the ships can be approximated by the equation $d(t) = 8\cos\frac{\pi}{6}t + 12$, where *t* is time, in hours, after the first high tide. (Berth: Sufficient space for a ship to maneuver; sea room)

a) Graph the function for two cycles starting at $t = 0$.

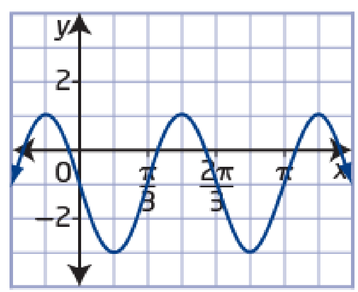
b) What is the period of the tide?

c) An ocean liner requires a minimum of 13 m of water to dock safely. From the graph, determine the number of hours per cycle the ocean liner can safely dock.

d) If the minimum depth of the berth occurs at 6 hours, determine the depth of the water. At what other times is the water level at a minimum?

Determining an Equation Given a Graph

Example: The graph at right shows the function $y = f(x)$



a) Write the equation of the function in the form $y = a\sin b(x - c) + d$, $a > 0$.

b) Write the equation of the function in the form $y = a\cos b(x - c) + d$, $a > 0$.

Assignment: Page 250 #15ac, 16ac, 19a, 20, 24abcd

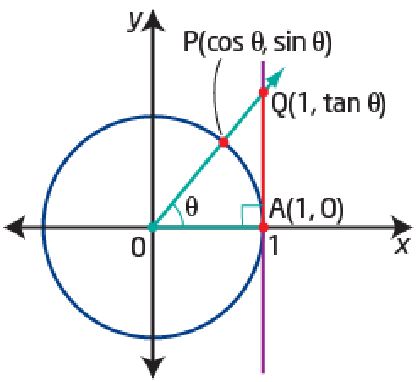
5.3 TheTangent Function (Day 1)

- A **tangent line to a curve** is a line that touches a curve, or a graph of a function, at a single point.

- The value of the **tangent of an angle** θ is the **slope of the line passing through the origin and the point on the unit circle** $(\cos\theta, \sin\theta)$. You can think of it as the **slope of the terminal arm** of angle θ in standard position.

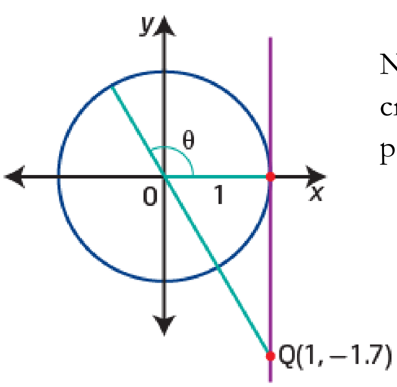
  If $slope = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$ (change in "*y*" divided by change in "*x*"), then the slope of the terminal arm of an angle in standard position is expressed by $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

- The tangent ratio is the **length of the line segment** that is tangent to the unit circle at the point $A(1,0)$, from the *x*-axis to the terminal arm of angle θ at point Q.



From the diagram, the distance AQ is equal to the *y*-coordinate of point Q. Therefore, point Q has coordinates $(1, \tan\theta)$.

Example: Determine $\tan\theta$ and the value of θ, in degrees.



Note that the angle actually terminates in QII. They extended it downward to create the intersection with the tangent line to the unit circle, and stated the values of point Q.

$\tan\theta =$

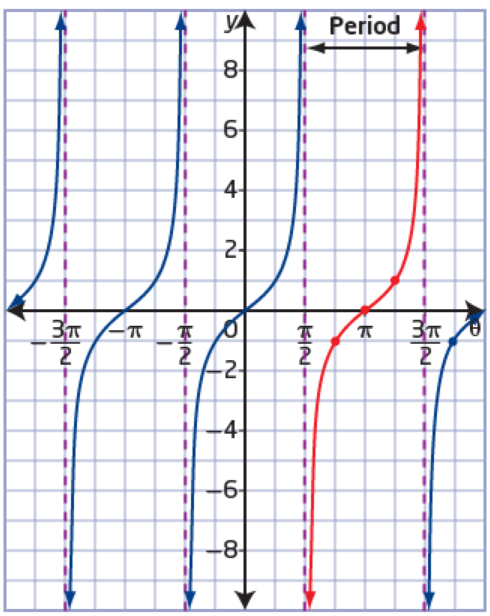
Now find the value of θ in degrees.

The Graph of the Tangent Function, $y = \tan\theta$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
sinθ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tanθ																	

What does $y = \tan\theta$ look like?

Between asymptotes, the graph of $y = \tan\theta$ passes through a point with y-coordinate -1, a θ-intercept, and a point with y-coordinate 1.



- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph $y = \tan x$, draw one asymptote; draw the points where $y = -1$, $y = 0$, and $y = 1$; and then draw another asymptote.
- The tangent function $y = \tan x$ has the following characteristics:
  - The period is π.
  - The graph has no maximum or minimum values.
  - The range is $\{y \mid y \in \mathrm{R}\}$.
  - Vertical asymptotes occur at $x = \frac{\pi}{2} + n\pi$, $n \in \mathrm{I}$.
  - The domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$.
  - The x-intercepts occur at $x = n\pi$, $n \in \mathrm{I}$.
  - The y-intercept is 0.

<u>Assignment</u>: Page 262 #1 - 3

 <u>5.3 TheTangent Function</u> (Day 2)

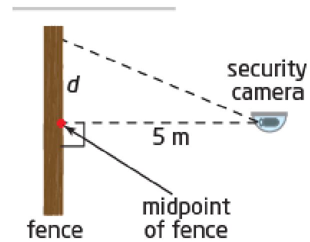
<u>Applications of the Tangent Function</u>

- **Slowly read through Example 2, p. 260. Breathe.**

<u>Together:</u> p. 264 #9

Security camera scans a long, straight fence. Camera is mounted on a post that is 5 m from the midpoint of the fence. Camera makes **one complete rotation in 60s**.

a) Determine the tangent function that represents the distance, *d*, in metres, along the fence from its midpoint as a function of time, *t*, **in seconds**, if the camera is aimed at the midpoint of the fence at *t = 0*.



b) **Sketch** the function in the interval $-15 \le t \le 15$.

c) What is the distance from the midpoint of the fence at $t = 10s$, to the nearest 10th of a m?

d) **Describe** what happens when $t = 15s$.

<u>Hmwk</u>: p. 264 #10 - 12

5.4 Equations and Graphs of Trigonometric Functions

A. Solving Trigonometric Equations

Example: Determine the solutions for the trigonometric equation $2\cos^2 x - 1 = 0$ for the interval $0^\circ \leq x \leq 360^\circ$. (After solving, see graph on page 268)

Example: Determine the **general solution** for the trigonometric equation $16 = 6\cos\frac{\pi}{6}x + 14$. Express answer to the nearest hundredth. (After solving, see graphs on page 269)

B. Applications of Trigonometric Functions

Trigonometric functions are frequently used to model real life occurrences that are cyclic or periodic such as water depth due to tides, population growth, electricity, sound waves, etc. They can also be used to represent shapes that look like the curve such as roller coasters, water slides, etc.

Example: The population of water buffalo is given by $P(t) = 250\sin\frac{\pi}{2}t + 400$ where t is the number of years since the first estimate was made.

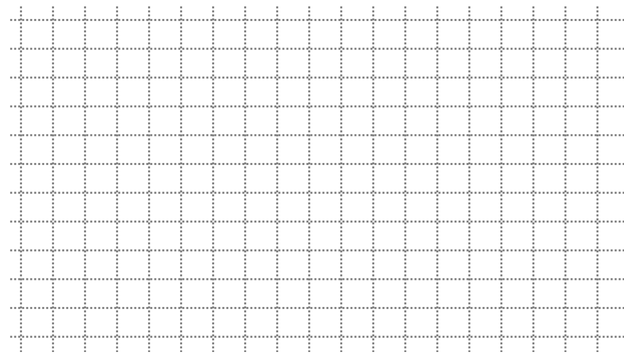
a) What was the initial estimate?

b) What was the population after 2 years?

5.4 Equations and Graphs of Trigonometric Functions (Day 2)

Ex. A carnival Ferris wheel with a radius of 7 m makes one complete revolution every 60 seconds. The bottom of the wheel is 1.5 m above the ground.

a) Draw the graph to show how a person's height about the ground varies with time.

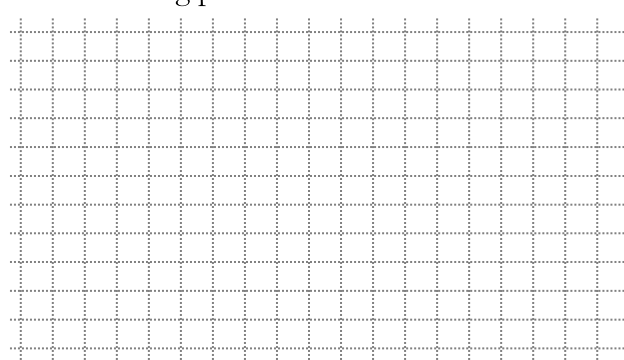


b) Find the equation of the graph in (a) starting at the beginning of the ride.

c) Find the equation of the graph in (a) using sine.

Ex. A roof in the shape of an upside down cosine wave is to be built to cover an arena. The arena is 24 m wide. The height at either side wall is 5 m and the maximum height is 9 m.

a) Sketch the graph using the height of the left wall as the starting point.



b) Find a function that gives the height of the roof in terms of the distance from the left wall.

c) Find the height of the roof 10 m from the left wall.

d) Find the distance from the left wall when the height of the roof is 8 m.

5.4 Equations and Graphs of Trigonometric Functions

Applications of Trigonometric Functions

Trigonometric functions are frequently used to model real life occurrences that are cyclic or periodic such as water depth due to tides, population growth, electricity, sound waves, etc. They can also be used to represent shapes that look like the curve such as roller coasters, water slides, etc.

Example: Page 276 #6a

Domain =

Range =

Example: The population of water buffalo is given by $P(t) = 250\sin\frac{\pi}{2}t + 400$ where t is the number of years since the first estimate was made.

a) What was the initial estimate?

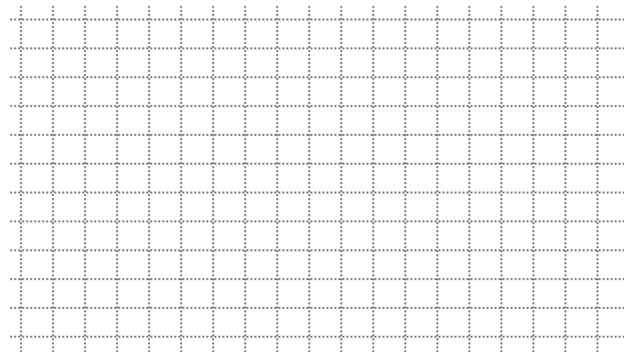
b) What was the population after 2 years?

c) What is the period of his function?

d) Find the smallest population size and when it first occurs. Sketch one cycle.

<u>Example:</u> A carnival Ferris wheel with a radius of 7 m makes one complete revolution every 60 seconds. The bottom of the wheel is 1.5 m above the ground.

a) Draw the graph to show how a person's height about the ground varies with time.



b) Find the equation of the graph using sine.

.

c) What is the height after 50 seconds?

<u>Assignment</u>: Page 276 #6, 9, 10, 12, 15ab, 19ab