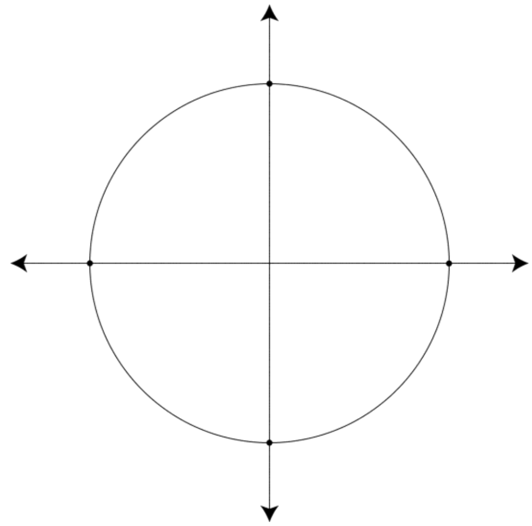
Pre-Calculus 30

4.1 Angles and Angle Measure



Angles can be measured in ***degrees*** or ***radians.*** Angle measures without units are considered to be in radians.

Radian: One radian is the measure of the central angle subtended in a circle by an arc equal in length to the radius of the circle.
(1 radian ≈ ________ degrees)

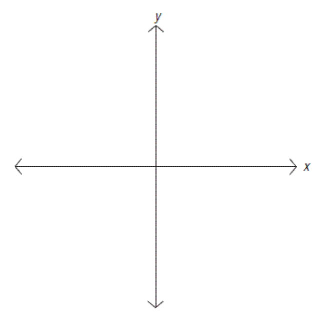
One full rotation of a circle is ________ degrees or ______ radians

One half rotation of a circle is ________ degrees or ______ radians

To convert degrees to radians, since $\frac{2\pi}{360^{\circ}} = \frac{\pi}{180^{\circ}} = 1$, multiply by $\frac{\pi}{180}$

To convert radians to degrees, since $\frac{360^{\circ}}{2\pi} = \frac{180^{\circ}}{\pi} = 1$, multiply by $\frac{180}{\pi}$

Angles in Standard Position



- vertex is at ________________________
- initial arm lies on the _______________ x-axis
- positive angle rotates ________________________
- negative angle rotates ________________________

Example: Convert each degree measure to radians (in reduced fractional form) and each radian measure to degrees. Draw each angle in standard position.

a) 150^0 = ______ radians

b) -270^0 = _______ radians

c) $\frac{7\pi}{6}$ = _________ degrees

d) -1.2 radians = _________ degrees

<u>Coterminal Angles</u> – angles in standard position with the same terminal arm. They may be measured in degrees or radians.

<u>Example</u>: Determine one positive and one negative angle that is coterminal with each angle. Draw the angles.

a) 40^0 b) -210^0 c) $\frac{\pi}{3}$

In general, angles *coterminal* with any angle θ can be described by:

$\theta \pm (360^0)n$ *or* $\theta \pm 2\pi n$ (n is a natural number)

<u>Example</u>: Write an expression for all possible angles coterminal with each given angle. Identify the angles that are coterminal that satisfy $-360^0 \leq \theta < 360^0$ or $-2\pi \leq \theta < 2\pi$

a) 110^0 b) -500^0 c) $\frac{9\pi}{4}$

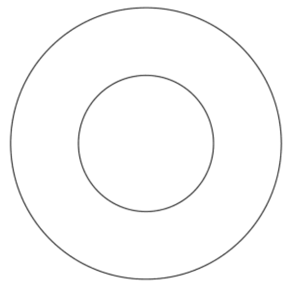
<u>Assignment</u>: Page 175 #1 – 5 (ace), 6 bde, 7abc, 9ab, 11abd

Pre-Calculus 30 **4.1 Arc Length of a Circle**

All arcs that subtend angle θ have the same central angle, but have different arc lengths depending on the radius of the circle. The ***arc length*** is proportional to the radius.

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}} \qquad \frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{sector angle}}{2\pi}$$

Consider the 2 concentric circles to the right with center at O. The radius of the smaller circle is 1 and the radius of the larger circle is r. Let *x* represent the arc length of the smaller circle and *a* is the arc length of the larger circle.



Using proportions:

$$\frac{a}{x} = \frac{r}{1} \qquad \frac{x}{2\pi r} = \frac{\theta}{2\pi}$$

$$a = xr \qquad x = \frac{\theta(2\pi(1))}{2\pi} \quad \text{since radius} = 1$$

$$x = \theta$$

So $a = \theta r$

a = θr	where *a* = arc length, *r* = radius, *θ* = angle measured in radians (*a* and *r* must be in the same units)

Example 4: Page 173

Your Turn: Page 174

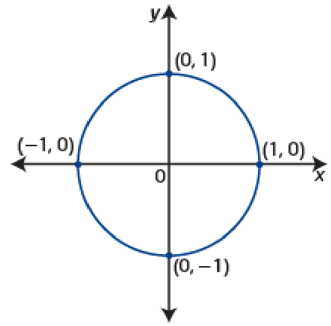
Assignment: Page 176 #12 – 15, 17

Pre-Calculus 30

4.2 The Unit Circle (Day 1)

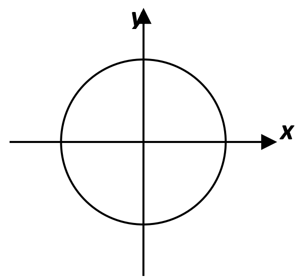
Unit Circle

- center at the origin
- radius of 1 unit
- equation of the unit circle is $x^2 + y^2 = 1$



Your Turn – Page 182

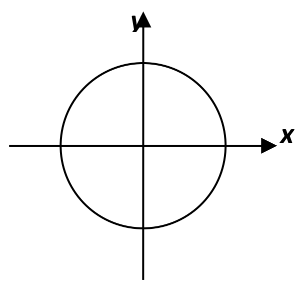
Find the equation of the circle with center at the origin and a radius of 6.



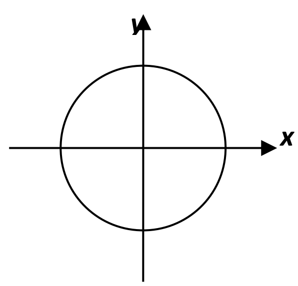
Page 183 – Example 2

Determine the coordinates for all points on the unit circle that satisfy the conditions. Draw a diagram.

a) *x* coordinate is $\frac{2}{3}$



b) *y* coordinate is $\frac{-1}{\sqrt{2}}$ and the point is in quadrant III



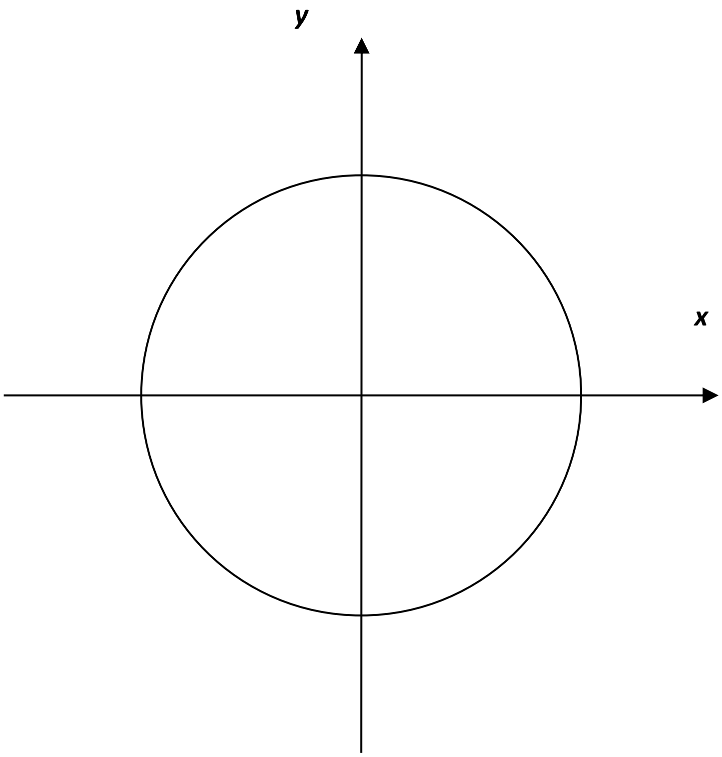
Assignment: Page 186 # 1ad, 2ace, 3

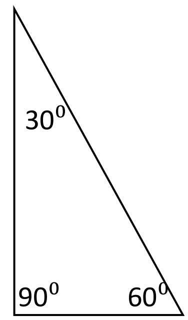
Pre-Calculus 30

4.2 The Unit Circle (Day 2)

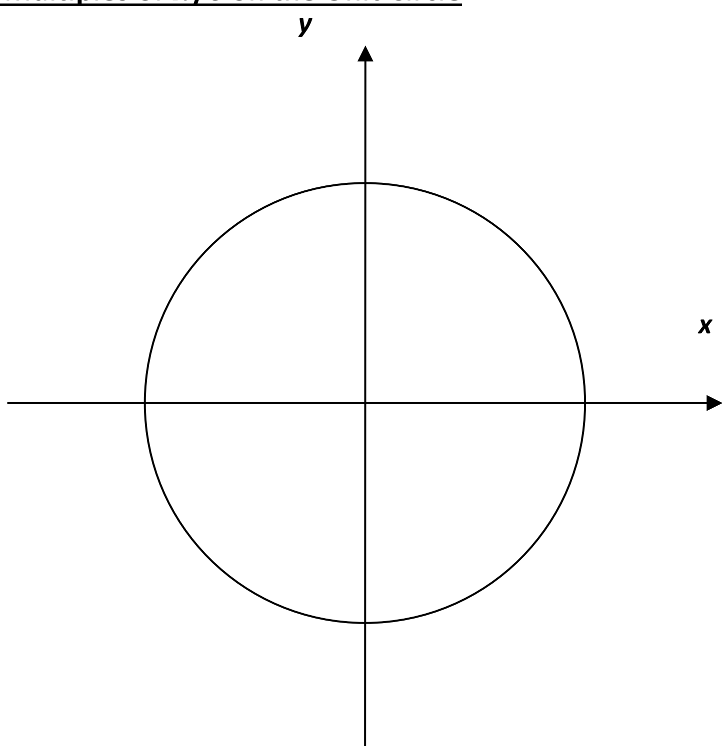
In the unit circle $a = \theta r$ becomes $a = \theta$ since $r = 1$. Therefore the central angle and its subtended arc have the same numerical value.

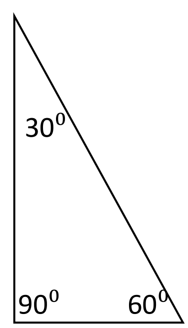
A. Multiples of $\pi/3$ on the Unit Circle



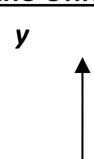


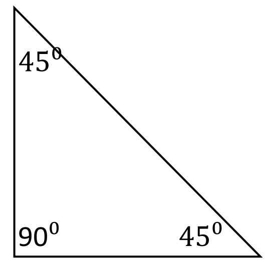
B. Multiples of $\pi/6$ on the Unit Circle

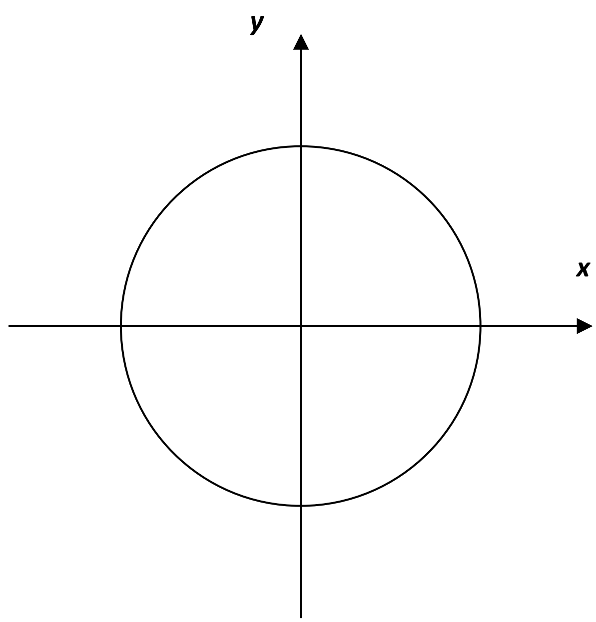




C. Multiples of $\pi/4$ on the Unit Circle







Summary:

$$P\left(\frac{\pi}{3}\right)=\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$$

$$P\left(\frac{\pi}{4}\right)=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$$

$$P\left(\frac{\pi}{6}\right)=\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$$

Look for patterns in the ordered pairs.

Note: If $P(\theta)=(a,b)$, then $P(\theta \pm 2\pi)=$

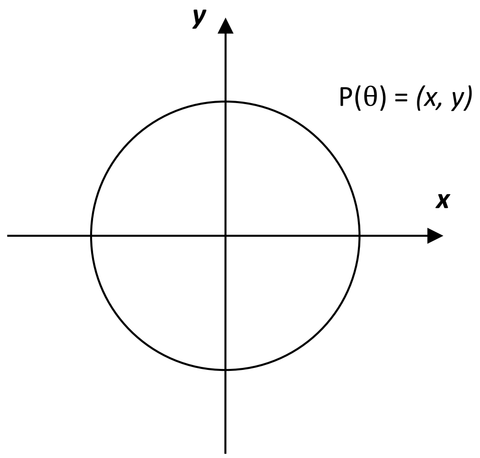
If $P(\theta)=(a,b)$, then $P(\theta \pm \pi)=$

If $P(\theta)=(a,b)$, then $P\left(\theta \pm \frac{\pi}{2}\right)=$

Assignment: Page187 #4 – 5 (acegi), 6, 9, 13, 19, C3

4.3 Trigonometric Ratios

Show how the coordinates of P(θ) can be represented by (cos θ, sin θ) for any θ in the unit circle. (This is #19 on page 190)



$\cos\theta = \frac{adj}{hyp}$ $\sin\theta = \frac{opp}{hyp}$

And $\tan\theta = \frac{y}{x} =$

Reciprocal Trigonometric Ratios

$csc\theta =$ $sec\ \theta =$ $cot\ \theta =$

Your Turn – Page 194

The point B $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

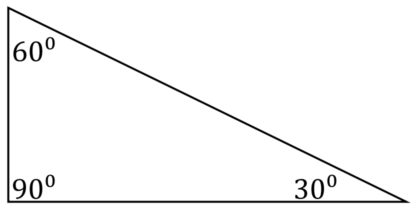
a) Draw a diagram to model the situation.

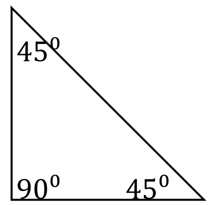
b) Determine the values of the 6 trig ratios for θ. Express answers in lowest terms.

Exact Values for Trig Ratios

Exact values for trig ratios can be found using the $30^0 - 60^0 - 90^0$ and the $45^0 - 45^0 - 90^0$ triangles along with the reference angle. Recall that a reference angle (θ_R) is a ***positive acute angle*** formed between the terminal arm of an angle in standard position and the nearest *x*-axis. The sign (+ or –) will be determined by the quadrant in which the terminal arm lies (use "CAST" or "All Soup Turns Cold")

A trigonometric ratio has an exact value if the given angle has a reference angle of 30^0, 45^0 or 60^0 (or is a quadrantal angle).





Example 2 – Page 194

Determine the exact value for each. Draw diagrams.

a) $\cos\frac{5\pi}{6}$

b) $\sin\left(-\frac{4\pi}{3}\right)$

c) $\sec 315^\circ$

d) $\cot 270^\circ$

Assignment: Page 201 #1acegi, 3, 6, 9abcd

Pre-Calculus 30 **4.3 Trigonometric Ratios**

Approximate Values of Trig Ratios

- use your calculator and set to "degrees" or "radians"
- to find the value of a reciprocal trig ratio (csc θ, sec θ, cot θ), use the definition of reciprocal

Example 3 – Page 196

Give the approximate value of each trig ratio. Round answers to 4 decimal places.

a) $\tan\frac{7\pi}{5}$ b) $\cos 260^0$ c) sin 4.2 d) $\csc(-70^0)$

Finding Angles, Given Their Trigonometric Ratios

- Use the appropriate inverse trig function key ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$). Your calculator will give just one answer ($0^0 \leq \theta \leq 90^0$)
- Use the reference angle in the correct quadrant to find the other angle(s)

If $\sin\theta = 0.5$, find θ in the domain $0^0 \leq \theta < 360^0$. Give exact answer.

Example: Determine the measure of all angles that satisfy each of the following. Use diagrams to help you find all possible answers.

a) $\cos\theta = 0.843$ in the domain $0^\circ \leq \theta < 360^\circ$. Give approximate answer to the nearest tenth.

b) $\sin\theta = 0$ in the domain $0^\circ \leq \theta \leq 180^\circ$. Give exact answers.

c) $\cot\theta = -2.777$ in the domain $-\pi \leq \theta < \pi$ (*radians). Give approximate answer to the nearest tenth.

d) $\csc\theta = -\frac{2}{\sqrt{2}}$ in the domain $-2\pi \leq \theta < \pi$ (*radians). Give exact answers.

Example 5 (page 200)

The point (-4, 3) lies on the terminal arm of an angle θ in standard position. What is the exact value of each trig ratio for θ?

Assignment: Page 202 #2, 8, 10 – 12(abc)

4.4 Introduction to Trig Equations

Interval Notation

- $0 < \theta < \pi$ can be written as $\theta \in (0, \pi)$
- $0 \le \theta < \pi$ can be written as $\theta \in [0, \pi)$
- $0 \le \theta \le \pi$ can be written as $\theta \in [0, \pi]$

Examples – Solve each trig equation in the specified domain. Give solutions as exact values where possible. Otherwise, give approximate angle measures to the nearest thousandth of a radian.

1. $5\sin\theta + 2 = 1 + 3\sin\theta, \quad 0 \le \theta < 2\pi$

2. $3\csc x - 6 = 0, \quad x \in [0^\circ, 360^\circ)$

3. $\tan^2\theta - 5\tan\theta + 4 = 0, \quad \theta \in [0, 2\pi)$

4. $\sin^2 x - 1 = 0, \quad 0 \le x < 2\pi$

Assignment: Page 211 #1, 3ac, 4abf, 5ace, 6, 7, 8, 10