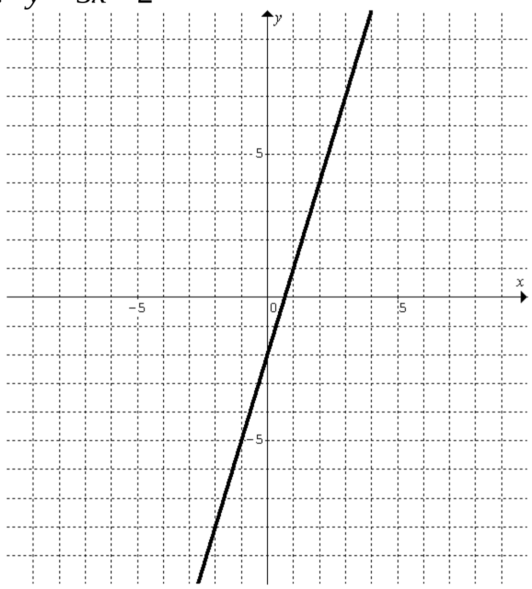
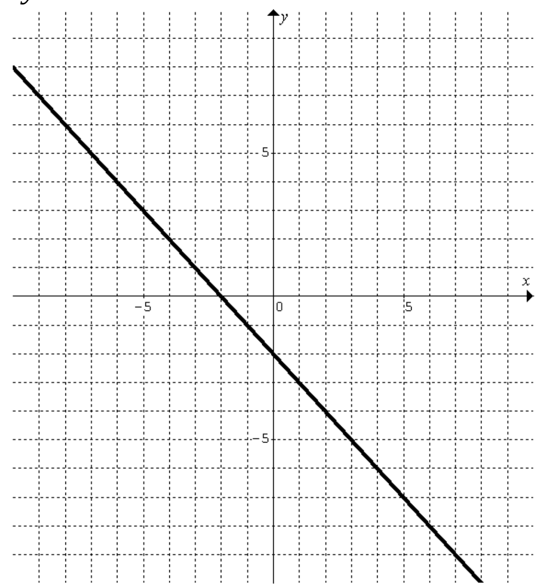
Pre-Calculus 30 **3.1 Characteristics of Polynomial Functions – End Behaviour (Day 1)**

For each function, identify the ***coefficient*** and the ***degree of the leading term***. Indicate the quadrants in which each graph begins and ends.

1. $y = 3x - 2$

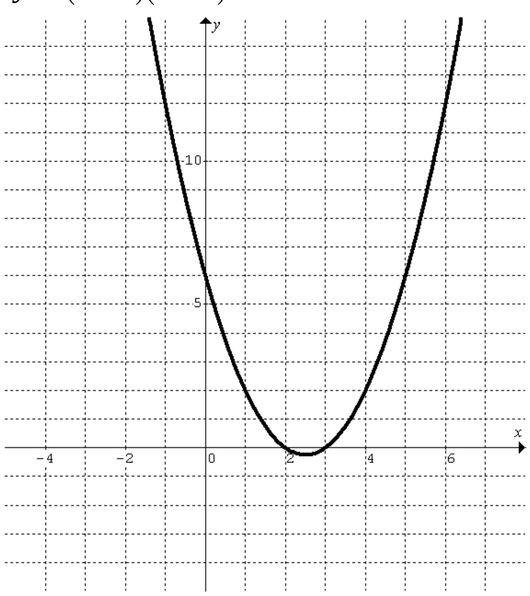


2. $y = -x - 2$



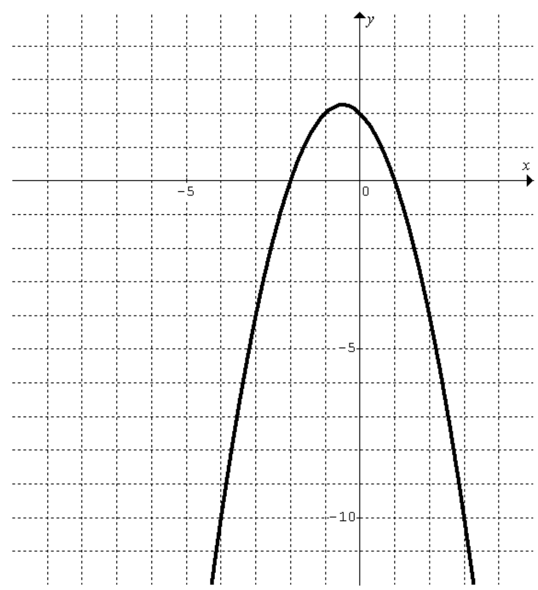
3. $y = x^2 - 5x + 6$

$y = (x-3)(x-2)$



4. $y = -x^2 - x + 2$

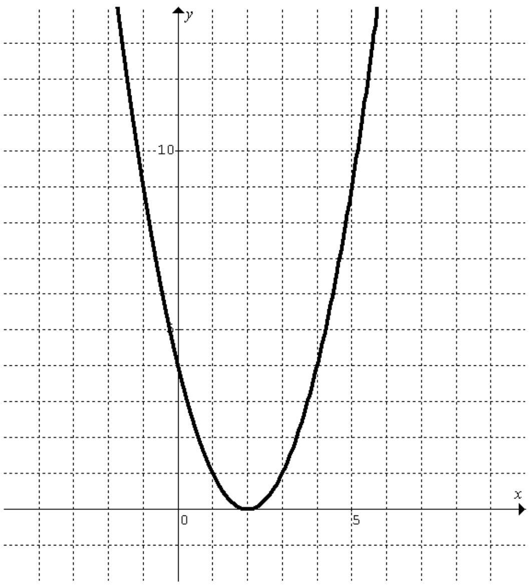
$y = -(x-1)(x+2)$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
1.				
2.				
3.				
4.				

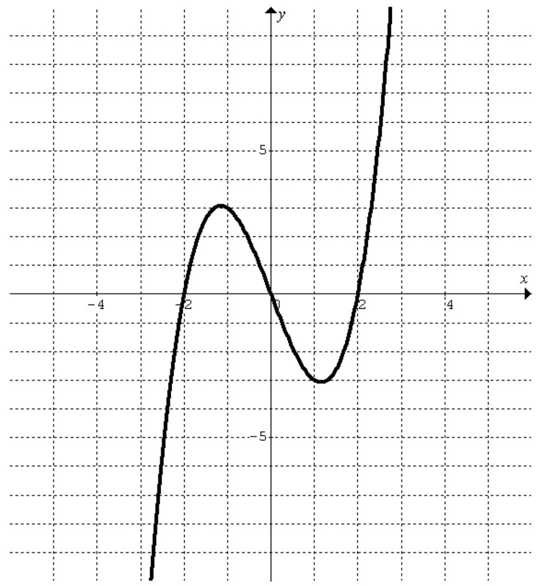
5. $y = x^2 - 4x + 4$

$y = (x-2)^2$



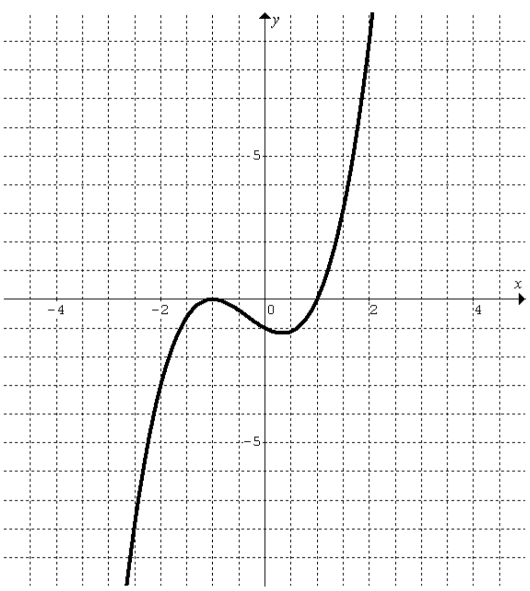
6. $y = x^3 - 4x$

$y = x(x-2)(x+2)$



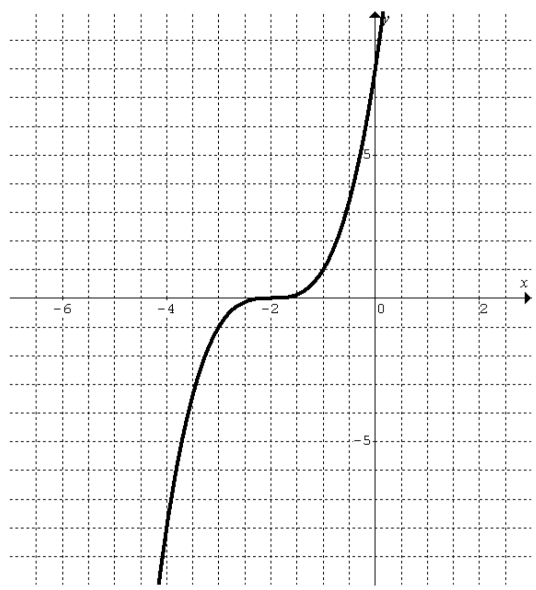
7. $y = x^3 + x^2 - x - 1$

$y = (x+1)^2(x-1)$



8. $y = x^3 + 6x^2 + 12x + 8$

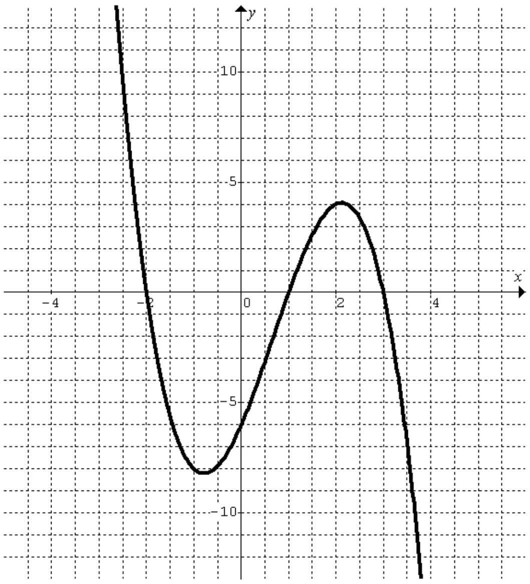
$y = (x+2)^3$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
5.				
6.				
7.				
8.				

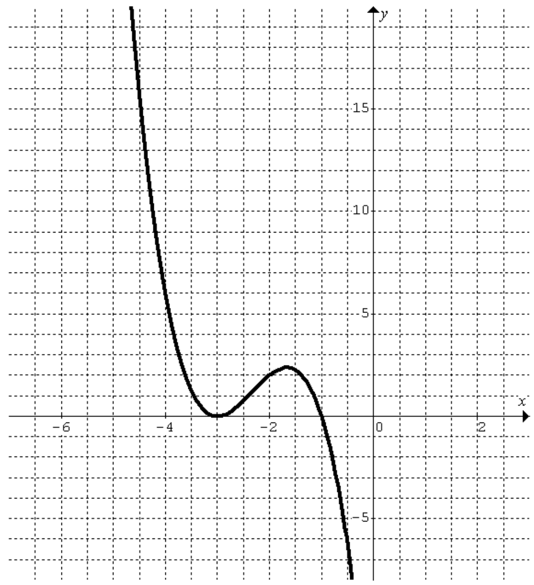
9. $y=-x^3+2x^2+5x-6$

$y=-(x+2)(x-1)(x-3)$



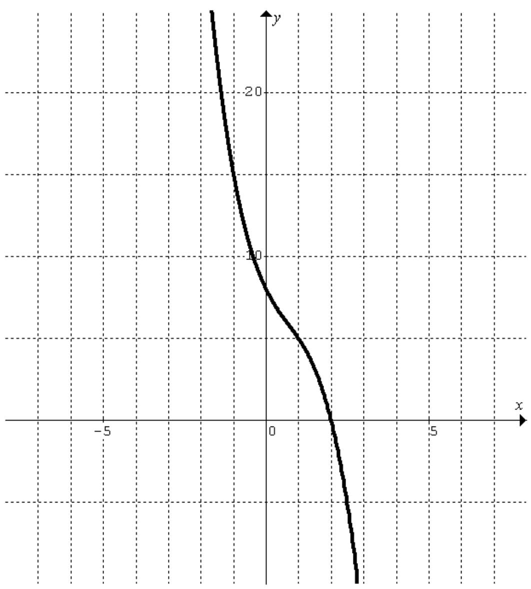
10. $y=-2x^3-14x^2-30x-18$

$y=-2(x+1)(x+3)^2$



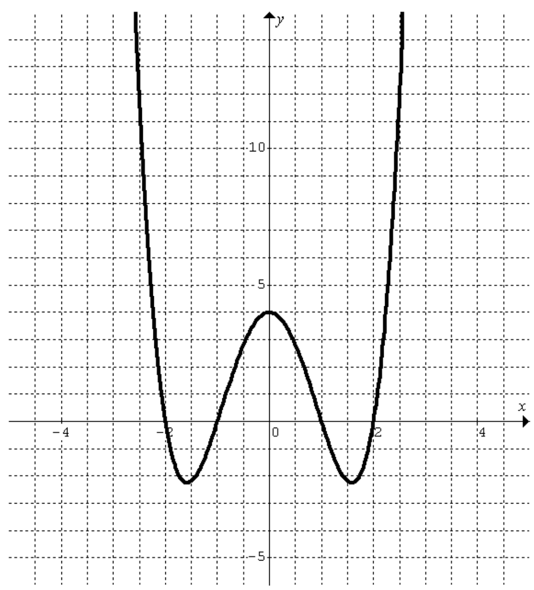
11. $y=-x^3+2x^2-4x+8$

$y=-(x^2+4)(x-2)$



12. $y=x^4-5x^2+4$

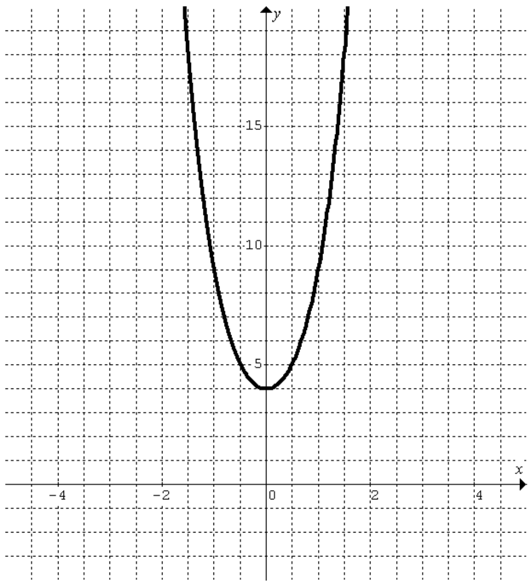
$y=(x+1)(x-1)(x+2)(x-2)$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
9.				
10.				
11.				
12.				

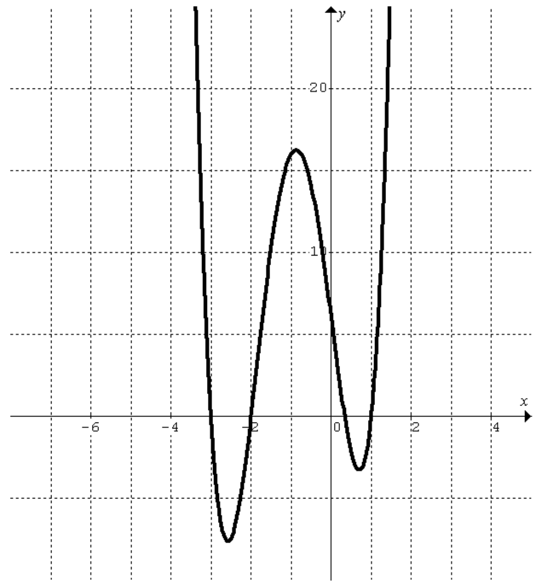
13. $y = x^4 + 4x^2 + 4$

$y = (x^2 + 2)^2$



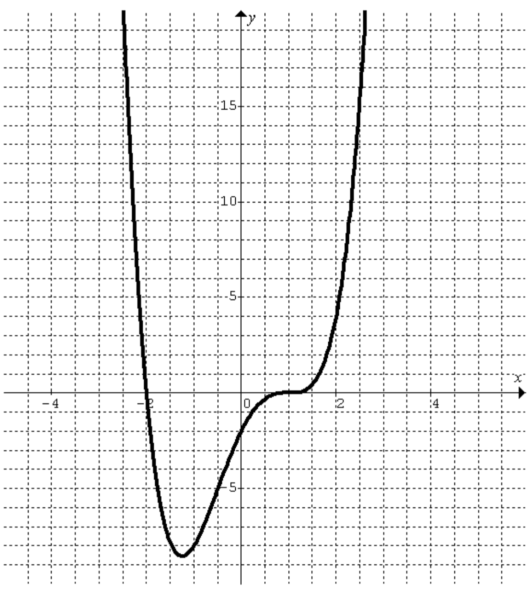
14. $y = 3x^4 + 11x^3 - x^2 - 19x + 6$

$y = (x + 2)(x + 3)(3x - 1)(x - 1)$



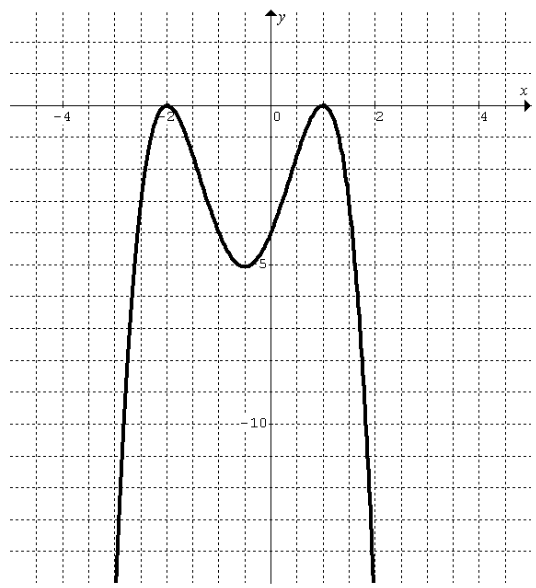
15. $y = x^4 - x^3 - 3x^2 + 5x - 2$

$y = (x + 2)(x - 1)^3$



16. $y = -x^4 - 2x^3 + 3x^2 + 4x - 4$

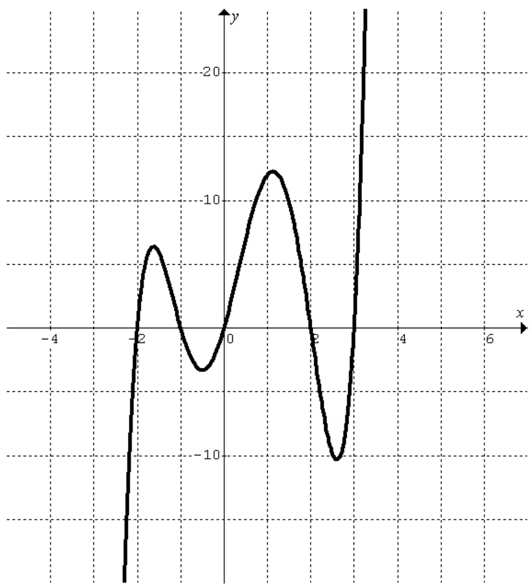
$y = -(x + 2)^2 (x - 1)^2$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
13.				
14.				
15.				
16.				

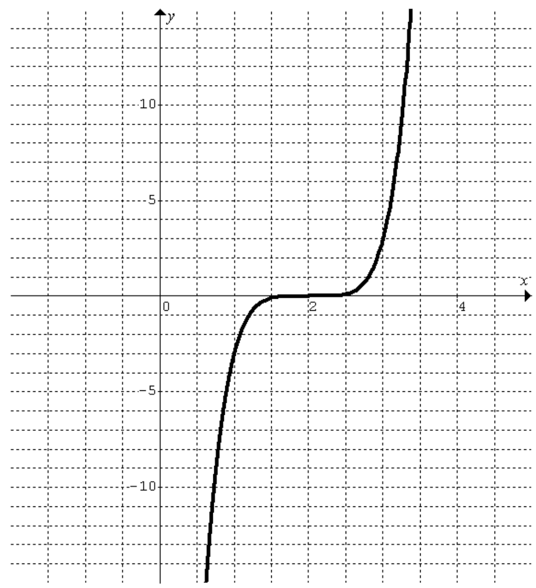
17. $y = x^5 - 2x^4 - 7x^3 + 8x^2 + 12x$

$y = x(x+1)(x+2)(x-2)(x-3)$



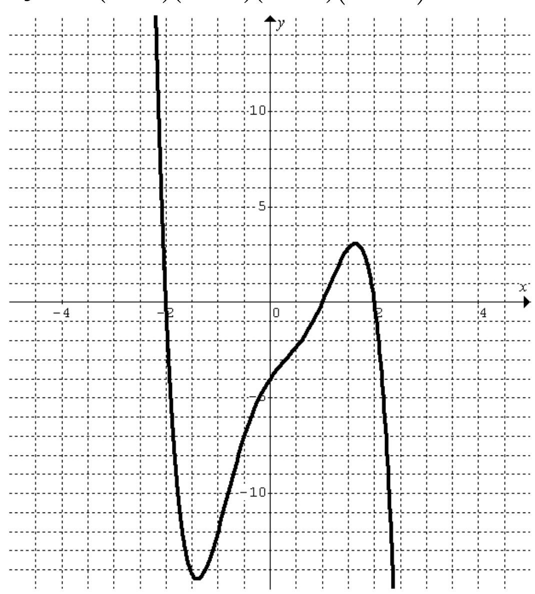
18. $y = 3x^5 - 30x^4 + 120x^3 - 240x^2 + 240x - 96$

$y = 3(x-2)^5$



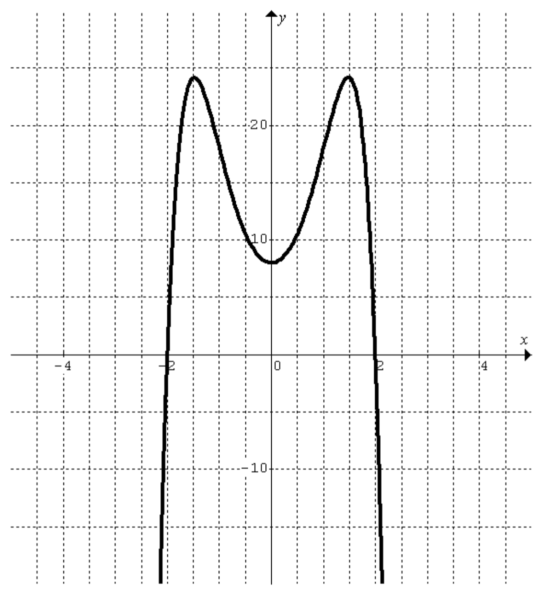
19. $y = -x^5 + x^4 + 3x^3 - 3x^2 + 4x - 4$

$y = -(x-1)(x-2)(x+2)(x^2+1)$



20. $y = -x^6 + x^4 + 10x^3 + 8$

$y = -(x-2)(x+2)(x^2+1)(x^2+2)$



	Leading term		Quadrant in which graph:	
	coefficient	degree	begins	ends
17.				
18.				
19.				
20.				

<u>Observations</u>:

1. Look at the leading term for each function. Which functions have an odd degree and a positive leading coefficient?

2. Based on your observations in question 1, the graph of a polynomial function with an odd degree and a positive leading coefficient will begin in quadrant ____ and end in quadrant ____.

3. Which functions have an odd degree and a negative leading coefficient?

4. Based on your observations in question 3, the graph of a polynomial function with an odd degree and a negative leading coefficient will begin in quadrant ____ and end in quadrant ____.

5. Which functions have an even degree and a positive leading coefficient?

6. Based on your observations in question 5, the graph of a polynomial function with an even degree and a positive leading coefficient will begin in quadrant ____ and end in quadrant ____.

7. Which functions have an even degree and a negative leading coefficient?

8. Based on your observations in question 7, the graph of a polynomial function with an even degree and a negative leading coefficient will begin in quadrant ____ and end in quadrant ____.

9. What is the domain of all of these functions?

A relationship exists between the number of possible x-intercepts and the degree of the polynomial function.

- **Odd degree**: Minimum of one x-intercept; maximum number of x-intercepts is indicated by the degree of the polynomial function
- **Even degree**: Minimum of zero x-intercepts; maximum number of x-intercepts is indicated by the degree of the polynomial function

Example: A polynomial function of degree of 6 may have no x-intercepts or as many as 6 x-intercepts.
A polynomial function of degree of 5 may have one x-intercept or as many as 5 x-intercepts.

<u>Assignment:</u> Page 114 #3

3.1 Characteristics of Polynomial Functions (Day 2)

A polynomial function is a function of the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots a_2x^2 + a_1x^1 + a_0$ where:

n is a whole number, x is a variable, and the coefficients a_n to a_0 are real numbers. (See page 108)

- The ***degree*** **(n)** of a polynomial function is the ***greatest exponent*** of the variable x that exists in the equation of the function.
- The ***coefficient*** of the greatest power of x is the ***leading coefficient***. The text book calls this term a_n.
- The ***constant term*** represents the $y-$intercept of the function. The text book calls this term a_0.
- In our course, the coefficients are restricted to being ***integral values*** (integers).
- ***End Behaviour*** – the behaviour of the y-values of a function as $|x|$ becomes very large.

Names of Polynomials Functions

- Degree 0 – Constant Function
- Degree 1 – Linear function
- Degree 2 – Quadratic function
- Degree 3 – Cubic function
- Degree 4 – Quartic function
- Degree 5 – Quintic function

Page 110 – Example 2 (Match functions to graphs)

Page 112 – Example 3 (Application)

Pre-Calculus 30 **3.1 Summary of Characteristics of Polynomial Functions**

A polynomial function has the form…..

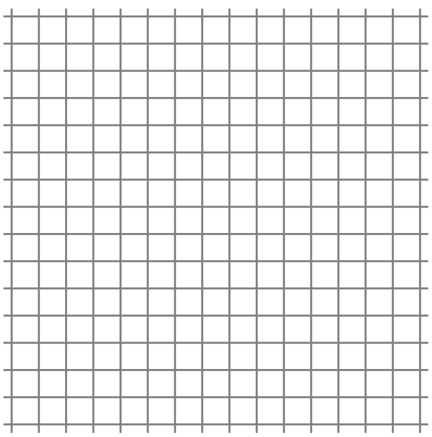
1. Odd-Degree Polynomials

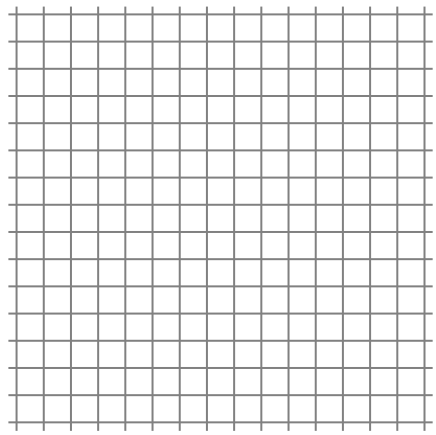
a) ***Positive*** leading coefficient

Begins in: Ends in:

b) ***Negative*** leading coefficient

Begins in: Ends in:



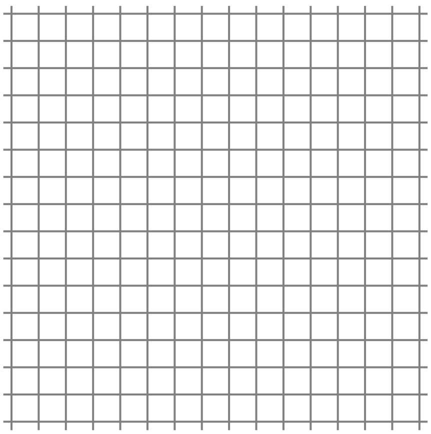


- y-intercept:
- x-intercept:
- domain:
- range:

2. Even-Degree Polynomials

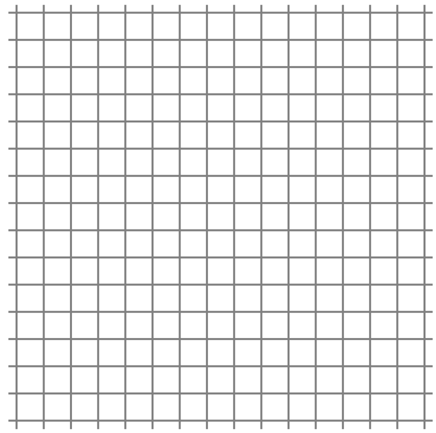
a) ***Positive*** leading coefficient

Begins in: Ends in:



b) ***Negative*** leading coefficient

Begins in: Ends in:



- y-intercept:
- x-intercept:
- domain:
- range:

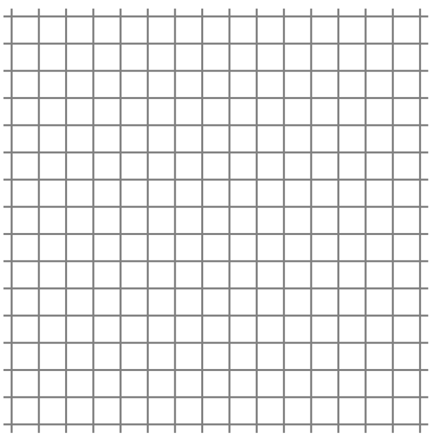
3.1 Summary of Characteristics of Polynomial Functions

A polynomial function has the form…..

3. Odd-Degree Polynomials

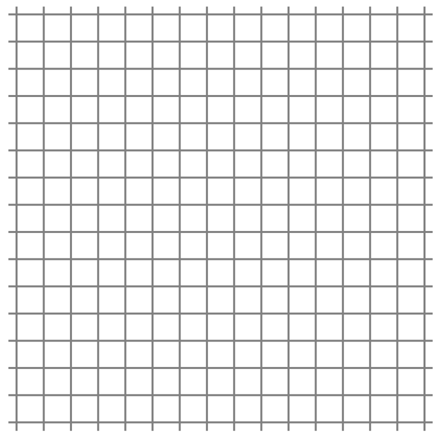
a) ***Positive*** leading coefficient

Begins in: Ends in:



b) ***Negative*** leading coefficient

Begins in: Ends in:



- y-intercept:
- x-intercept:
- domain:
- range:

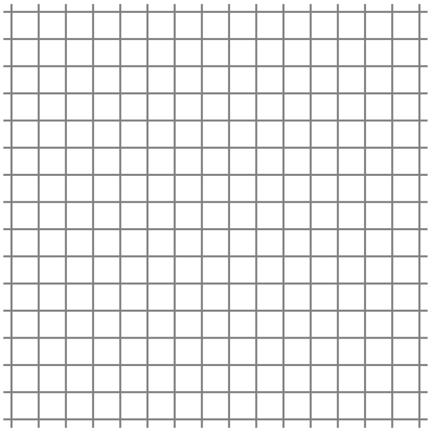
4. Even-Degree Polynomials

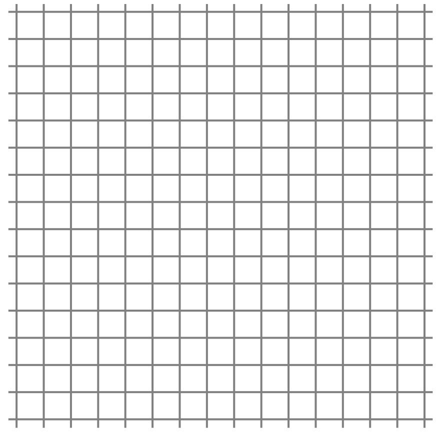
a) ***Positive*** leading coefficient

Begins in: Ends in:

b) ***Negative*** leading coefficient

Begins in: Ends in:





- y-intercept:
- x-intercept:
- domain:
- range:

3.1 Characteristics of Polynomial Functions

A polynomial function is a function of the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots a_2 x^2 + a_1 x^1 + a_0$ where:

n is a whole number, x is a variable, and the coefficients a_n to a_0 are real numbers. (See page 108)

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- Degree 5 – Quintic function

Page 110 – Example 2 (Match functions to graphs)

Page 112 – Example 3 (Application)

Assignment: Page 114 #1ace, 2ace, 3, 4ace, 6, 7ab, 9bcd

Pre-Calculus 30 **3.2 The Remainder Theorem**

Recall Long Division – divisor, dividend, quotient, remainder

1. $8\overline{)4218}$

2. $x+3\overline{)x^2+7x+17}$

Synthetic Division - a quicker way to perform polynomial long division

When dividing a polynomial by *(x – a),* we will write

$$\begin{array}{l|l} \underline{a} & \text{coefficients of polynomials} \\ \hline & \text{quotient} \qquad \boxed{\text{remainder}} \end{array}$$

Steps: 1. Write the value of "*a*" in upper left hand box

2. Across the top, record coefficients of the dividend in descending order. Use zero for any coefficient of missing powers.

3. Bring down the first coefficient

4. Multiply the first coefficient by "*a*" and record it under the second coefficient. Then add these numbers together.

5. Continue doing this until the last coefficient is done. This is now your remainder.

6. Write the answer as $polynomial + \dfrac{remainder}{x-a}$

7. Restrictions – the denominator cannot equal zero…therefore $x - a \neq 0$

Examples:

1. $(x^2 + 7x + 17) \div (x + 3)$

2. $(x^3 + 7x^2 - 3x + 4) \div (x - 2)$

Remainder Theorem

When a polynomial *P(x)* is divided by a binomial *(x – a)*, the remainder is *P(a)*.

Example:

Use the Remainder Theorem to find the remainder when $P(x) = x^3 - 10x + 6$ is divided by $x + 4$.

Now use synthetic division to verify this.

Assignment: Page 124 #4, 5abc, 6abc, 7abc, 8ab, 11a, 12a

Pre-Calculus 30 **3.3 The Factor Theorem**

EXPLORE

- Using the Remainder Theorem, find the remainder when $x^3 + 2x^2 - 5x - 6$ is divided by $x + 1$

- Determine the remainder $\dfrac{x^3+2x^2-5x-6}{x+1}$ using synthetic division.

- Factor the quotient portion of the answer above.

- Now write $x^3 + 2x^2 - 5x - 6$ as a product of its three factors.

- What do you notice about the remainder when you divide $x^3 + 2x^2 - 5x - 6$ by any of its three factors?

Factor Theorem : $(x - a)$ is a factor of a polynomial $P(x)$, if and only if $P(a) = 0$…or if the remainder when dividing is zero.

Example: Is $(x - 3)$ a factor of $x^3 - 3x^2 - x + 3$? Is $(x + 3)$ a factor?

Integral Zero Theorem: If $(x - a)$ is a factor of the polynomial function $P(x)$ with integral coefficients, then a is a factor of the constant term of $P(x)$.

Example: Factor $x^4 - 5x^3 + 2x^2 + 20x - 24$ fully. List possible factors.

Example: Factor $x^4 - 3x^3 - 7x^2 + 15x + 18$ completely.

Assignment: Page 133 #2ace, 3ace, 5ace, 6ace [7ac, 11, 13]

Pre-Calculus 30 **3.4 The Zeros of a Polynomial Function (Day 1)**

***y*-intercept** – the value at which the graph crosses the y-axis

- the value of y when $x = 0$

***x*-intercept(s)** – the value(s) at which the graph crosses the x-axis

- the value of x when $y = 0$. Also called the zeros of the function

multiplicity (of a zero) – the number of times a zero of a polynomial function occurs.

Use the graphs from the 3.1 handout to complete the chart. For **"behavior of the graph at the zero",** you have 3 options: **crosses**, **tangent to but does not cross** (**tt but dnc**), and **tangent to and crosses** (**tt and crosses**)

	constant term	y-intercept	algebraic factors of the polynomial	multiplicity	x-intercept(s) or zeros	behaviour of the graph at the zero
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						

	constant term	y-intercept	algebraic factors of the polynomial	multiplicity	x-intercept(s) or zeros	behaviour of the graph at the zero
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						

How can you identify the ***y*-intercept** of the function?

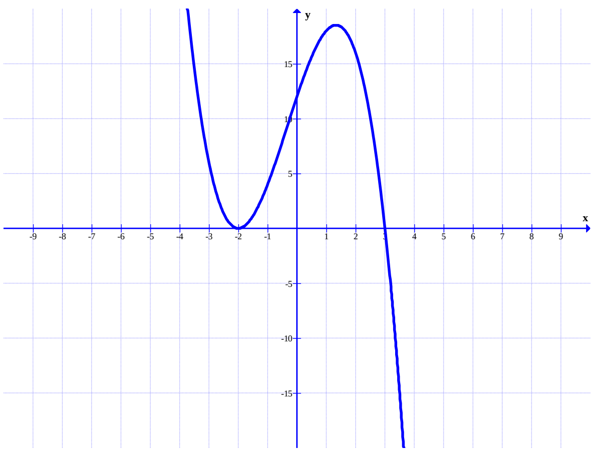
What is the **relationship** between the multiplicity of a factor and the behaviour of the graph at the corresponding zero (or x-intercept)? **Multiplicity of 1:** **Multiplicity odd but $\neq 1$:** **Multiplicity even:**

Ex.1, p. 138 For each graph, determine

- The least possible degree
- The sign of the leading coefficient
- The x-intercepts and the factors of the function (with least possible degree)
- The intervals where the function is positive (above the x-axis) and intervals where it is negative (below the x-axis)

Your Turn

For the following graph, determine the characteristics as listed above.



- The least possible degree

- The sign of the leading coefficient

- The x-intercepts and the factors of the function (with least possible degree)

- The intervals where the function is positive (above the x-axis) and intervals where it is negative (below the x-axis)

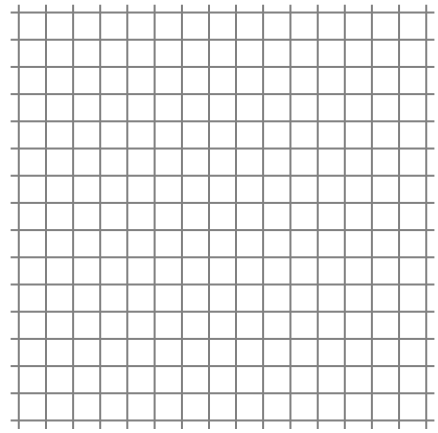
Assignment: Page 148 #4

Pre-Calculus 30 **3.4 The Zeros of a Polynomial Function (Day 2)**

Sketching Graphs of Functions: Equation Already Factored

Ex. Sketch the graph of $y=(x-1)(x+2)(x+3)$.

x-intercepts:

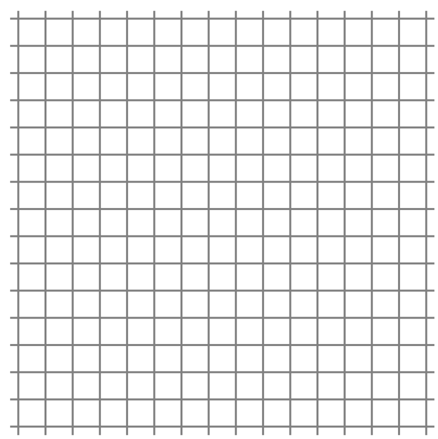


Degree:
Leading coefficient:
Begins in ____ and ends in ____
y-intercept:

'reference points' to add detail to graph:

Ex. Sketch the graph of $f(x)=-(x+2)^3(x-4)$

x-intercepts:



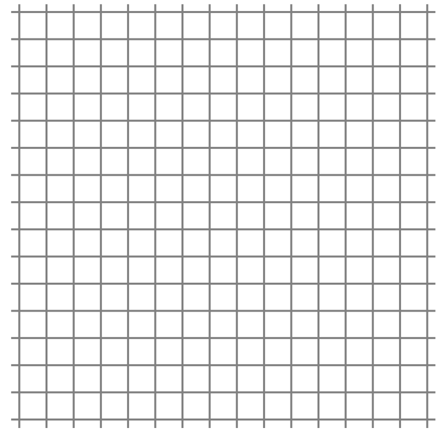
Degree:
Leading coefficient:
Begins in ____ and ends in ____

y-intercept:
'reference points' to add detail to graph:

Sketching Graphs of Functions: Equation Not Yet Factored

<u>Ex.</u> Sketch the graph of $y = -2x^3 + 6x - 4$

Factor First!



x-intercepts:

Degree:
Leading coefficient:
Begins in ____ and ends in ____
y-intercept:
'reference points' to add detail to graph:

<u>Assignment:</u> Page 149 #3, 9e, 7/8, 10

Pre-Calculus 30 **<u>3.4 The Zeros of a Polynomial Function</u> (Day 3)**

<u>Applying Transformations to Sketch a Graph</u>

- $y = a(b(x-h))^n + k$
- Use the key points of the base functions $y = x^3$, $y = -x^3$, $y = x^4$ $y = -x^4$ and use **mapping** to find the points of the new, transformed function.

<u>Ex.</u> The graph of $y = x^3$ is transformed to obtain the graph of $y = -2\left(4\left(x-1\right)\right)^3 + 3$. State the parameters and use mapping to find the new points. Sketch the graph.

$a =$ $b =$ $h =$ $k =$

For $y = x^3$, mapping of key points looks like:

$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

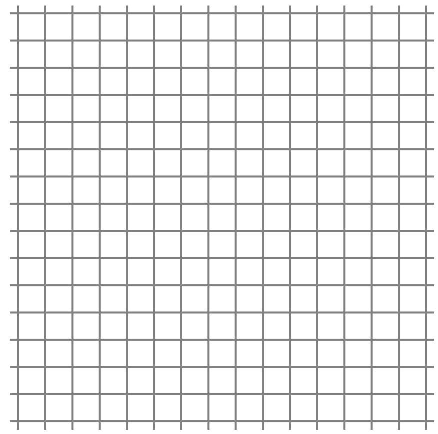
$(-2, \quad) \rightarrow$

$(-1, \quad) \rightarrow$

$(0, \quad) \rightarrow$

$(1, \quad) \rightarrow$

$(2, \quad) \rightarrow$



<u>Ex. 4, p. 145</u> Applications –Algebraic Solution only

- A "Let statement" is required.
- A sentence is required at the end, **not a solution set**.

<u>Ex.</u> Determine the cubic equation with zeros -3 (multiplicity of 2) and 2, and *y*-intercept -18.

<u>Assignment</u>**:** page 148 #3, 6c (mapping), 13, 14ab, 16

Pre-Calculus 30 **3.4 The Zeros of a Polynomial Function (Day 1)**

***y*-intercept** – the value at which the graph crosses the *y*-axis

- the value of y when $x = 0$

***x*-intercept(s)** – the value(s) at which the graph crosses the *x*-axis

- the value of x when $y = 0$. Also called the zeros of the function

multiplicity (of a zero) – the number of times a zero of a polynomial function occurs.

Using the graphs from the 3.1 handout, complete the chart below. For the **"behavior of the graph at the zero",** you have **3 options**: crosses, tangent to but does not cross (TNC), and tangent to and crosses (TC)

	constant term	*y*-intercept	algebraic factors of the polynomial	multiplicity	*x*-intercept(s) or zeros	behaviour of the graph at the zero
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						

	constant term	y-intercept	algebraic factors of the polynomial	multiplicity	x-intercept(s) or zeros	behaviour of the graph at the zero
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						

How can you identify the ***y*-intercept** of the function?

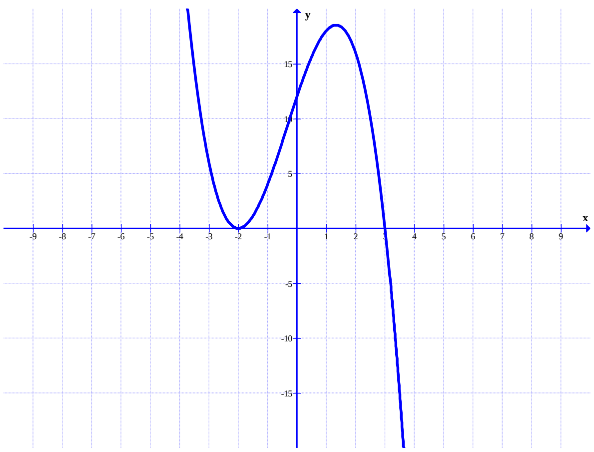
What is the **relationship** between the multiplicity of a factor and the behaviour of the graph at the corresponding zero (or x-intercept)? **Multiplicity of 1:** **Multiplicity odd but $\neq 1$:** **Multiplicity even:**

Ex.1, p. 138 For each graph, determine

- The least possible degree
- The sign of the leading coefficient
- The *x*-intercepts and the factors of the function (with least possible degree)
- The intervals where the function is positive (above the *x*-axis) and intervals where it is negative (below the *x*-axis)

Your Turn

For the following graph, determine the characteristics as listed above.



- The least possible degree

- The sign of the leading coefficient

- The *x*-intercepts and the factors of the function (with least possible degree)

- The intervals where the function is positive (above the *x*-axis) and intervals where it is negative (below the *x*-axis)

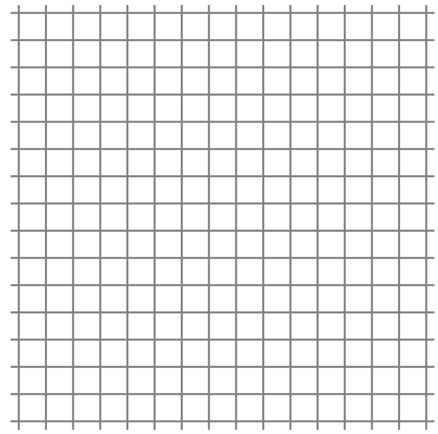
Assignment: Page 148 #4

3.4 The Zeros of a Polynomial Function **(Day 2)**

Sketching Graphs of Functions: Equation Already Factored

Ex. Sketch the graph of $y=(x-1)(x+2)(x+3)$.

x-intercepts:



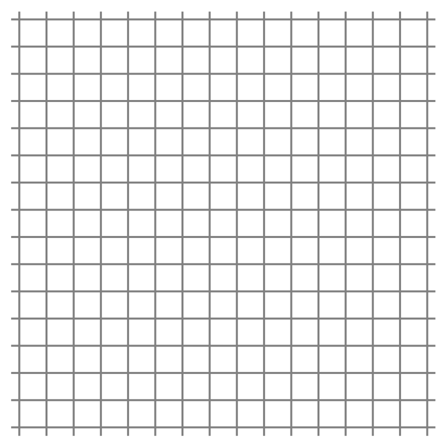
(also discuss multiplicity and behavior)

Degree:
Leading coefficient:
Begins in ____ and ends in ____
y-intercept:

'reference points' to add detail to graph:

Ex. Sketch the graph of $f(x)=-(x+2)^3(x-4)$

x-intercepts:



(also discuss multiplicity and behavior)

Degree:
Leading coefficient:
Begins in ____ and ends in ____

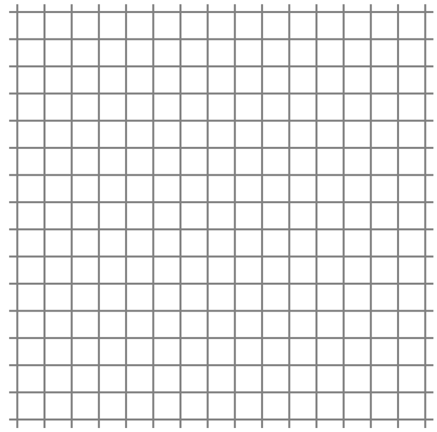
y-intercept:

'reference points' to add detail to graph:

Sketching Graphs of Functions: Equation Not Yet Factored

<u>Ex.</u> Sketch the graph of $y = -2x^3 + 6x - 4$

Factor!



x-intercepts:

(also discuss multiplicity and behavior)

Degree:
Leading coefficient:
Begins in ____ and ends in ____
y-intercept:

'reference points' to add detail to graph:

<u>Hmwk:</u> p. 149 #9e, then 8 (use our list of info to sketch the graphs of the equations in #7)

<u>3.4 The Zeros of a Polynomial Function</u> (Day 3)

<u>Applying Transformations to Sketch a Graph</u>

- a, b, h, k are back!
- Knowing the precise shape of base functions such as $y = x^3$, $y = -x^3$, $y = x^4$ and $y = -x^4$ is **essential**!
- You can use the key points of the base function and **use mapping** to find the points of the new, transformed function.

<u>Ex.</u> The graph of $y = x^3$ is transformed to obtain the graph of $y = -2\left(4(x-1)^3\right) + 3$. State the parameters, use mapping to find new points and sketch the graph.

$a =$ $b =$ $h =$ $k =$

For $y = x^3$, mapping of key points looks like:

$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

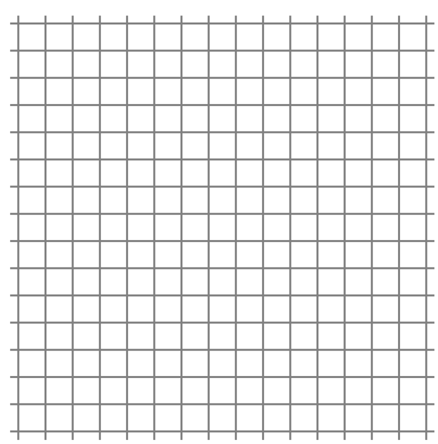
$(-2, \quad) \rightarrow$

$(-1, \quad) \rightarrow$

$(0, \quad) \rightarrow$

$(1, \quad) \rightarrow$

$(2, \quad) \rightarrow$



Ex. 4, p. 145 **Applications – Focus on Algebraic Solution**

- A "Let statement" is required.
- Note that they needed to use the quadratic formula.
- A sentence is required at the end, **not a solution set**.

Your Turn Three consecutive integers have a product of -210. What are the three integers?

Ex. Determine the cubic equation with zeros -3 (multiplicity of 2) and 2, and *y*-intercept -18.

Hmwk: p. 148 #3, 5, 6 (mapping), 10, 11, 13, 14, 16