1.1 Horizontal and Vertical Translations

Frieze Patterns

Transformation – a shifting or change in shape of a graph

Mapping – the relating of one set of points to another set of points (ie. points on the original graph and points on the transformed graph.)

Translation – moves the graph up, down, left or right. (the location of the graph changes, but the shape does not)

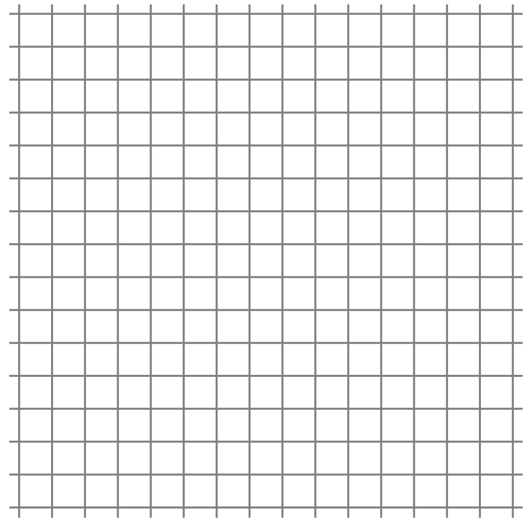
Image Points – the point that is the result of a transformation of a point on the original graph (Often a prime symbol next to the letter representing an image point is used. Ex. A′)

Investigate

1. a) Let $f(x) = |x|$ Use a table of values to compare the following outputs. $y = f(x)$, $y = f(x) + 3$, $y = f(x) - 3$, given input values of -3, -2, -1, 0 1, 2, 3

   b) Graph the functions on the same set of coordinate axes.

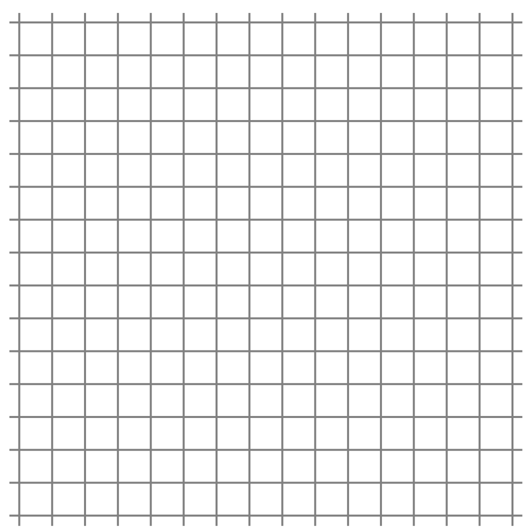
x	$f(x) = \|x\|$	$f(x) = \|x\| - 3$	$f(x) = \|x\| + 3$
-3			
-2			
-1			
0			
1			
2			
3			



2. a) Let $f(x) = |x|$ Use a table of values to compare the following outputs. $y = f(x)$, $y = f(x+3)$, $y = f(x-3)$, given input values of -3, -2, -1, 0 1, 2, 3

   b) Graph the functions on the same set of coordinate axes.

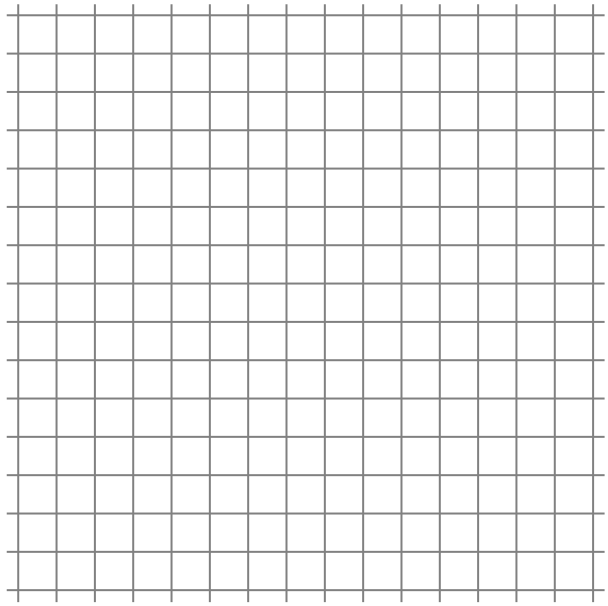
x	$f(x) = \|x\|$	$f(x) = \|x-3\|$	$f(x) = \|x+3\|$
-9			
-6			
-3			
0			
3			
6			
9			



3. a) Graph the functions $y = x^2$, $y - 2 = x^2$, *and* $y = (x-5)^2$ on the same set of coordinate axes.

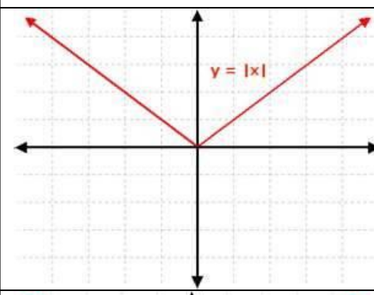
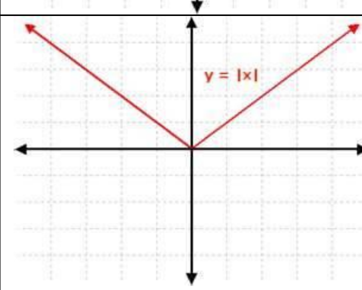
x	$y = x^2$	$y = x^2 + 2$
-3		

-2		
-1		
0		
1		
2		
3		



x	$y=(x-5)^2$
2	
3	
4	
5	
6	
7	
8	

Key Points

Function	Transformation from $y = f(x)$	Mapping	Visual Example
$y - k = f(x)$ or $y = f(x) + k$			
$y = f(x - h)$			

Pre-Calculus 30 1.2 Reflections and Stretches

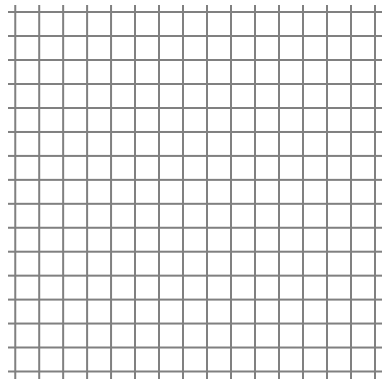
Reflection – a transformation where each point on the original graph has an image point resulting from a reflection in a line (orientation may change, but shape remains the same).

Invariant Point – a point on a graph that remains unchanged after a transformation is applied to it (it lies on the line of reflection)

Investigate: Reflections of a Function $y = f(x)$

1. Draw the graph of $y = f(x)$ using the given points. Graph $y = f(-x)$ on the same set of axes. Give the mapping notation of the key points representing the transformation.

$y=f(x)$		$y=f(-x)$
(x, y)	→	
(-6, 6)	→	
(-2, -2)	→	
(0, -2)	→	
(3, 1)	→	
(5, 0)	→	

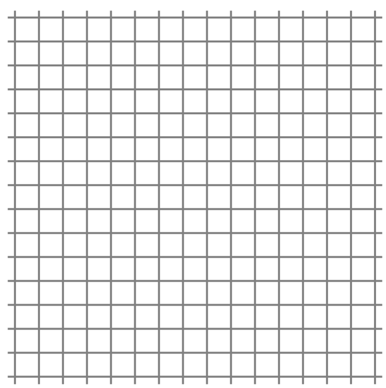


Invariant point(s):____________

$y = f(-x)$ represents a ___________________ reflection of the function in the __ - axis

2. Draw the graph of $y = f(x)$ using the given points. Graph $y = -f(x)$ on the same set of axes. Give the mapping notation of the key points representing the transformation.

$y=f(x)$		$y=-f(x)$
(x, y)	→	
(-6, 6)	→	
(-2, -2)	→	
(0, -2)	→	
(3, 1)	→	
(5, 0)	→	

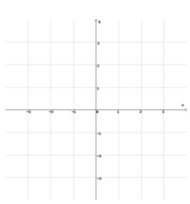


Invariant point(s):____________

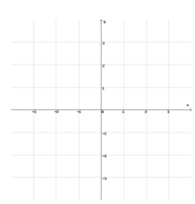
$y = -f(x)$ represents a ___________________ reflection of the function in the __ - axis

Summary :

Vertical Reflection
Function : $y = -f(x)$
Mapping: $(x, y) \rightarrow$
line of reflection: x-axis



Horizontal Reflection
Function : $y = f(-x)$
Mapping: $(x, y) \rightarrow$
line of reflection: y-axis

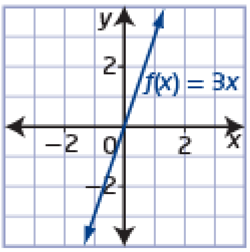


Assignment: Page 28 #1ab, 3, 4 (see below for #3 and #4)

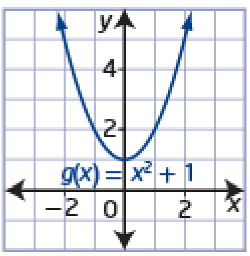
3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes.
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

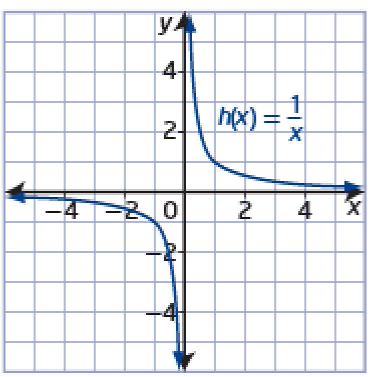
a)



b)



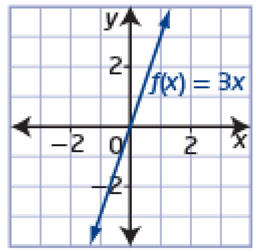
c)



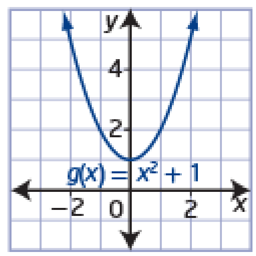
4. Consider each function in #3.

- Copy the graph of the function and sketch its reflection in the y-axis on the same set of axes.
- State the equation of the reflected function.
- State the domain and range for each function.

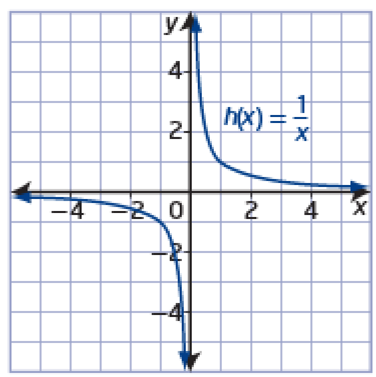
a)



b)



c)



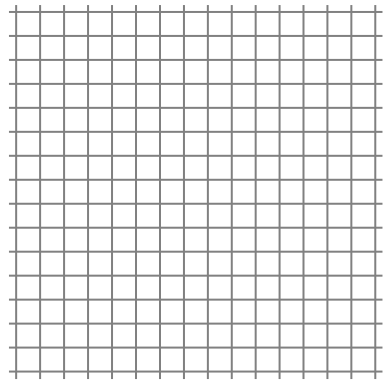
Pre-Calculus 30 <u>1.2 Reflections and Stretches (con't)</u>

<u>Stretch</u> – a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor (orientation does not change, but the shape changes).

1. <u>Vertical Stretches of a Function $y = af(x)$</u>
   - A **vertical stretching** is the stretching of the graph **away** from the *x*-axis. ($|a| > 1$)
   - A **vertical compression** is the squeezing of the graph **towards** the *x*-axis. ($|a| < 1$)
   - If *a* is **negative,** then the vertical compression or vertical stretching of the graph is **followed by a reflection** across the *x*-axis.

Complete the table below using the given *x* values. Then graph of *f(x)*, *g(x)* and *h(x)*.

x	$y = f(x)$	$y = g(x) = 2f(x)$	$y = h(x) = \frac{1}{2}f(x)$
-6	4		
-2	0		
0	2		
2	0		
6	4		



Invariant Point(s):

<u>For *f(x)*:</u>
Domain is
Range is

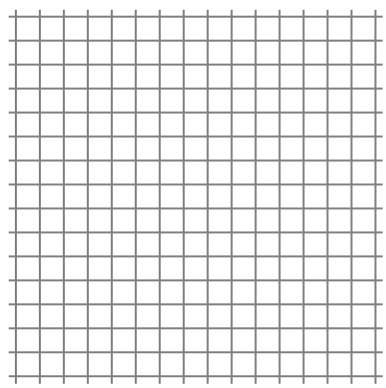
<u>For *g(x)*:</u>
Domain is
Range is

<u>For *h(x)*:</u>
Domain is
Range is

2. <u>Horizontal Stretches of a Function $y = f(bx)$</u>
   - A **horizontal stretching** is the stretching of the graph **away** from the ***y*-axis**.
   - A **horizontal compression** is the squeezing of the graph **towards** the ***y*-axis**.
   - If *b* is **negative**, the horizontal compression or horizontal stretching of the graph is **followed by a reflection** of the graph across the ***y***-axis.

Complete the table below, then graph *f(x)*, *g(x)* and *h(x)*

x	$y = f(x)$	x	$y = g(x) = f(2x)$	x	$y = h(x) = f\left(\frac{1}{2}x\right)$
-4	4	-2		-8	
-2	0	-1		-4	
0	2	0		0	
2	0	1		4	
4	4	2		8	



Invariant Point(s):

<u>For *f(x)*:</u>

<u>For *g(x)*:</u>

<u>For *h(x)*:</u>

Domain is
Range is

Domain is
Range is

Domain is
Range is

Summary:

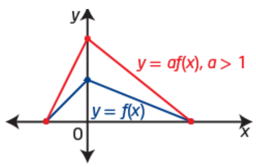
Vertical Stretch/Compression

$y = af(x)$

vertical stretch by a factor of $|a|$

If $a < 0$, graph is reflected in the x-axis

$(x, y) \longrightarrow (x, ay)$



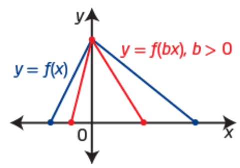
Horizontal Stretch/Compression

$y = f(bx)$

horizontal stretch by a factor of $\frac{1}{|b|}$

If $b < 0$, graph is reflected in the y-axis

$(x, y) \longrightarrow (\frac{x}{b}, y)$



Assignment: Page 28 #2ab, 6a, 7ac, 8, 9abcd

Pre-Calculus 30 **1.3 Combining Transformations**

Translation: $y = f(x-h)+k$ $(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$

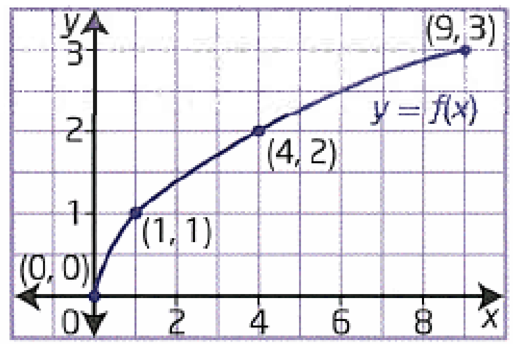
Reflection: $y = -f(x)$ $(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$

$y = f(-x)$ $(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$

Stretch: $y = af(bx)$ $(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$

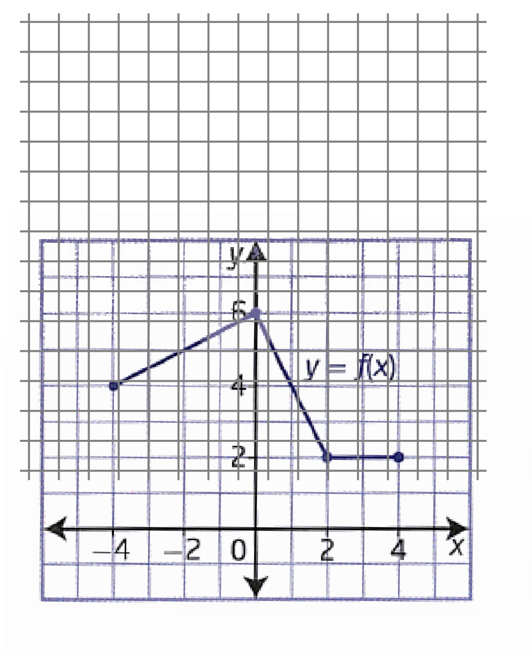
Your Turn for Example 1

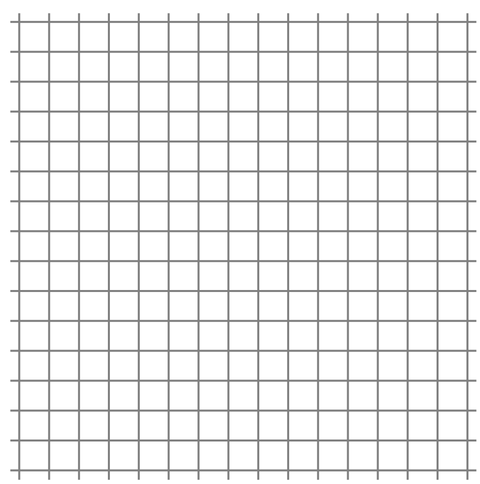
Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.



a) $y = 2f(x) - 3$

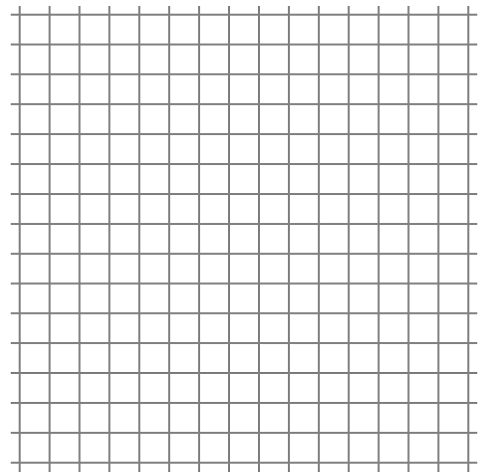
b) $y = f\left(\frac{1}{2}x - 2\right)$





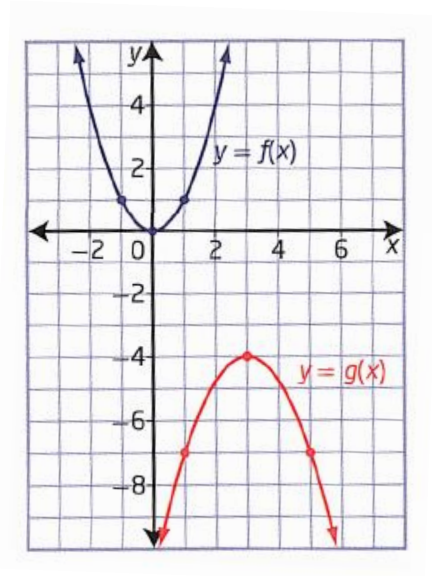
Your Turn for Example 2

Describe the combination of transformations that should be applied to the function $y = x^2$ to obtain the transformed function $g(x) = -2f\left(\frac{1}{2}(x+8)\right) - 3$. Sketch the graph, showing each step of the transformation. Write the corresponding equation and sketch the graph of $g(x)$.



Your Turn for Example 3

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. State the equation of the transformed function. Explain your answer.



1.3 Combining Transformations

- Write the function in the form: $y = a\ f\left(b\left(x-h\right)\right)+k$ in order to identify the transformations.
- Stretches and reflections can be done in any order but ***before*** translations.
- a corresponds to a ____________________stretch about the x-axis by a factor of $|a|$. If a is negative, the function is reflected in the ___________.
- b corresponds to a ____________________stretch about the y-axis by a factor of $\frac{1}{|b|}$. If b is negative, the function is reflected in the ______________.
- h corresponds to a ____________________translation
- k corresponds to a ____________________translation
- Mapping Notation: $(x,y)\rightarrow$

Ex. 1 Given the graph of $y = f\left(x\right)$, sketch the graph of:

a) $y=3f\left(2x\right)$

For this function, $a=$__, $b=$__, $h=$__, $k=$__ .

If each ordered pair of $y = f\left(x\right)$ is considered to be $\left(x,y\right)$, then the new ordered pairs will be $\left(\quad , \quad \right)$. So $\left(0,0\right)\rightarrow\left(\quad , \quad \right)$; $\left(1,1\right)\rightarrow\left(\quad , \quad \right)$; $\left(4,2\right)\rightarrow\left(\quad , \quad \right)$; $\left(9,3\right)\rightarrow\left(\quad , \quad \right)$.

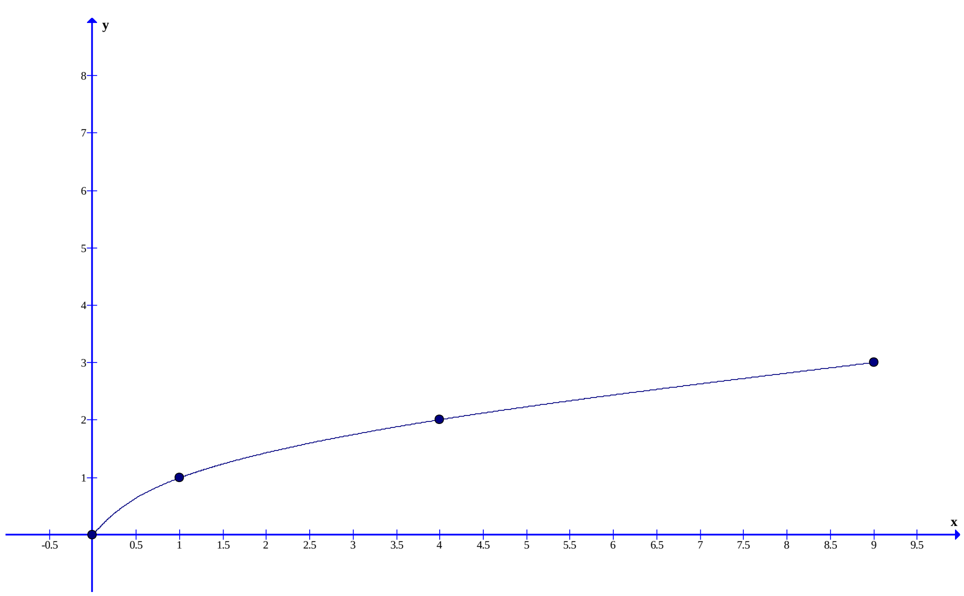
Sketch the new function and label it.

b) $y = f\left(3x+6\right)$

For this function, rewrite as $y = f\left(3\left(x+2\right)\right)$. Now $a=$__, $b=$__, $h=$__, $k=$__ .

If each ordered pair of $y = f\left(x\right)$ is considered to be $\left(x,y\right)$, then the new ordered pairs will be $\left(\quad , \quad \right)$.

So, $\left(0,0\right)\rightarrow\left(\quad , \quad \right)$; $\left(1,1\right)\rightarrow\left(\quad , \quad \right)$; $\left(4,2\right)\rightarrow\left(\quad , \quad \right)$, and $\left(9,3\right)\rightarrow\left(\quad , \quad \right)$. Sketch the new function and label it.



<u>Assignment</u>: Page 38 #2, 6, 9ace

<u>1.3 Combining Transformations – Writing Equations</u>

<u>Steps:</u>

1. Did the graph stretch vertically? $a =$

2. Did the graph stretch horizontally? $\frac{1}{b} =$ (**reciprocate to find *b* value)

3. Did it reflect? In which axis? Negate *a* and/or *b* value if necessary

4. Did the entire graph shift left/right? $h =$

5. Did the entire graph shift up/down? $k =$

6. Write equation in the form: $y = af(b(x-h))+k$

Stretches and Reflections must be done first. Then look at shifts up and down.

** Example 3 on page 37

** Page 38 #1a

<u>Assignment</u>: Page 38 #1b, 4, 8, 10b, 11ab

Pre-Calculus 30 **1.4 Inverse of a Relation**

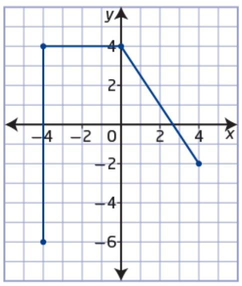
http://www.khanacademy.org/math/algebra/algebra-functions/v/introduction-to-function-inverses

Inverse of a Relation

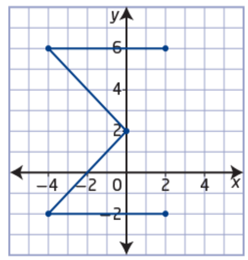
- Often thought of as "undoing" or "reversing" a position or effect
- found by interchanging the x-coordinates and y-coordinates of ordered pairs. *(x, y)* ⟶ *(y, x)*
- the domain and range of a relation become the range and domain, respectively, of the inverse of the relation.
- the graph of a relation and its inverse are *reflections* of each other in the line *y = x*
- the inverse of a function is not necessarily a function. (use *horizontal* line test on the original function to see if inverse is a function)
- when the inverse of a function *f(x)* is a function, it is written as $f^{-1}(x)$

Examples: Page 51 #2, 5e, 8a, 9c

#2a



#2b



Assignment: Page 52 #3abc, 5abcd, 7a, 8bc, 9b, 11, 13a, 15a, 16abc