

Sum of Functions

$$h(x) = f(x) + g(x) \quad \text{This can also be written as } h(x) = (f + g)(x).$$

Difference of Functions

$$h(x) = f(x) - g(x) \quad \text{This can also be written as } h(x) = (f - g)(x)$$

Examples

1. Given the functions $f(x) = 2x + 1$ and $g(x) = x^2$, determine the function $h(x) = (f + g)(x)$. Find $h(3)$.
2. Given the functions $f(x) = \sqrt{x-1}$ and $g(x) = x - 2$, determine $h(x) = f(x) - g(x)$.
3. If $h(x) = (f + g)(x)$ and $f(x) = 5x + 2$, determine $g(x)$ when $h(x) = \sqrt{x+7} + 5x + 2$.

Product of Functions

$$h(x) = f(x)g(x) \quad \text{This can also be written as } h(x) = (f \cdot g)(x)$$

Quotient of Functions

$$h(x) = \frac{f(x)}{g(x)} \quad \text{This can also be written as } h(x) = \left(\frac{f}{g}\right)(x), \quad \text{where } g(x) \neq 0$$

Examples Given $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$, determine the following and state any non-permissible values.

$$\text{a) } h(x) = (f \cdot g)(x) \qquad \text{b) } h(x) = \left(\frac{g}{f}\right)(x)$$

Assignment: Page 483 #1 – 4, 9 – 11 (just parts “a” and “c” of all questions)
Page 496 #1, 6 – 8 (just parts “a” and “c” of all questions)

10.3 Composite Functions (Day 1)

- We can substitute one function, $f(x)$, into another function, $g(x)$. The result would be $g(f(x))$. This is read “ g of f of x ”.
- The notation for this function composition is $(g \circ f)(x)$...**not to be confused with multiplication** which is $(g \bullet f)(x)$.

Example: Evaluate for $f(x) = 4x$, $g(x) = x + 6$, $h(x) = x^2$.

1. $f(g(3))$

2. $g(h(-2))$

3. $h(h(2))$

Example: If $f(x) = |x|$ and $g(x) = x + 1$, determine $f(g(-11))$.

Ex. 3, p. 503 Let $f(x) = x + 1$ and $g(x) = x^2$. Determine the equation of each composite function. Sketch the graph and state the domain and range.

a) $y = f(g(x))$

b) $y = f(f(x))$

10.3 Composite Functions (Day 2)**A. Composite functions and restrictions** – Does *order* matter when composing functions?

Consider $f(x) = \sqrt{x-1}$ and $g(x) = x^2$.

a) Determine $(f \circ g)(x)$.

b) Determine $(g \circ f)(x)$.

domain of $f(x)$:

domain of $g(x)$:

domain of $(f \circ g)(x)$

- find the restrictions of $g(x)$ and then of $f(g(x))$

domain of $(g \circ f)(x)$

- find the restrictions of $f(x)$ and then of $g(f(x))$

B. Determining the original functions from a composition.

If $h(x) = f(g(x))$, determine $f(x)$ and $g(x)$.

a) $h(x) = (x-2)^2 + (x-2) + 1$

Look for common elements. What do you see?

b) $h(x) = \sqrt{x^3 + 1}$

Start with the function inside the radical.

Your Turn If $h(x) = f(g(x))$, determine $f(x)$ and $g(x)$. Here, $h(x) = \sqrt[3]{x} + \frac{3}{3 + \sqrt[3]{x}}$.

C. Applications (Page 505 Example 5)

A spherical weather balloon is being inflated. The balloon's radius, r , in feet, after t minutes is given by $r = \sqrt{t}$.

a) Express the **volume** of the balloon as a function of time, t . Recall volume of a sphere is $V(r) = \frac{4}{3}\pi r^3$.

Compose the new function.

b) After how many minutes will the volume be 4000 ft³?

Your Turn. Same base question as above

a) Express the **surface area** of the balloon as a function of time, t . Recall surface area of a sphere is $SA = 4\pi r^2$

b) After how many minutes will the surface area be 180 ft²?

Assignment: Page 507 #6 – 8, 11a, 14, 16 (show 1 method only), 18

