Pre-Calculus 30 6.1 Reciprocal, Quotient, and Pythagorean Identities

Trigonometric Identity – a trig equation that is true for all permissible values of the variable in the expressions on both sides of the equation.

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

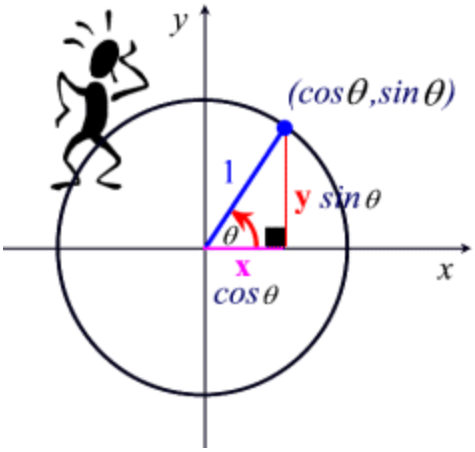
Example: a) Determine the non-permissible values, in degrees, for the equation $\sec\theta = \dfrac{\tan\theta}{\sin\theta}$

b) Verify that $\theta = 60^\circ$ is a solution of the equation

Example: Simplify the expression $\dfrac{\cot x}{\csc x \cos x}$

Pythagorean Theorem states:

$x^2 + y^2 = 1$ OR $\cos^2\theta + \sin^2\theta = 1$



1. Multiply this equation by $\frac{1}{\sin^2\theta}$

2. Multiply this equation by $\frac{1}{\cos^2\theta}$

Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$

Example: Verify the 3 Pythagorean Identities are true when $\theta = 30^\circ$

Assignment: Page 296 #1abd, 3 (simplify only), 4, 5a

Pre-Calculus 30 **6.3 Proving Identities**

Proving Identities – algebraically manipulate the sides of an equation into identical expressions. You may NOT perform operations across the equal sign when proving identities.

Strategies

- Use known identities to make substitutions (reciprocal identities)
- If the equation contains quadratics, use one of the Pythagorean Identities
- Rewrite the expression using only sine and cosine
- Multiply the numerator and denominator by the conjugate of an expression
- Factor to simplify expressions
- Manipulate the more complex side of the equation

Example: Prove $1-\sin^2 x = \sin x \cos x \cot x$

Example: Prove $\dfrac{\tan x \cos x}{\csc x} = 1-\cos^2 x$

Example: Prove $\dfrac{1-\cos x}{\sin x} = \dfrac{\sin x}{1+\cos x}$

Example: Prove $\csc^2 x + \csc^2 x \cot^2 x = \csc^4 x$

Assignment: Page 314 #1, 2, 3

Prove that each of the following equations is an identity. (Page 150 in Math C30 by Thiessen)